COMP 122/L Lecture 2

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Outline

- Operations on binary values
  - AND, OR, XOR, NOT
  - Bit shifting (left, two forms of right)
  - Addition
  - Subtraction
- Twos complement

Bitwise Operations

Bitwise AND

- Similar to logical AND (`&&`), except it works on a bit-by-bit manner
- Denoted by a single ampersand: `&`

```
(1001 &
 0101)=
 0001
```

Bitwise OR

- Similar to logical OR (`||`), except it works on a bit-by-bit manner
- Denoted by a single pipe character: `|`

```
(1001 |
 0101)=
 1101
```

Bitwise XOR

- Exclusive OR, denoted by a carat: ^
- Similar to bitwise OR, except that if both inputs are `1` then the result is `0`

```
(1001 ^
 0101)=
 1100
```

Bitwise NOT

- Similar to logical NOT (`!`), except it works on a bit-by-bit manner
- Denoted by a tilde character: `~`

```
~1001 =
 0110
```

Shift Left

- Move all the bits `N` positions to the left, subbing in `N` `0`s on the right

Shift Left

- Move all the bits `N` positions to the left, subbing in `N` `0`s on the right

`1001`

Shift Left

- Move all the bits `N` positions to the left, subbing in `N` `0`s on the right

```
1001 << 2 =
100100
```

Shift Left

- Useful as a restricted form of multiplication
- Question: how?

```
1001 << 2 =
100100
```

Shift Left as Multiplication

- Equivalent decimal operation:

```
234
```

Shift Left as Multiplication

- Equivalent decimal operation:

```
234 << 1 =
2340
```

Shift Left as Multiplication

- Equivalent decimal operation:

```
234 << 1 =
2340

234 << 2 =
23400
```

Multiplication

- Shifting left `N` positions multiplies by `(base)`^{N}
- Multiplying by 2 or 4 is often necessary (shift left 1 or 2 positions, respectively)
- Often a whooole lot faster than telling the processor to multiply
- Compilers try hard to do this

```
234 << 2 =
23400
```

Shift Right

- Move all the bits `N` positions to the right, subbing in **either** `N` `0`s or `N` `1`s on the left
  - Two different forms

Shift Right

- Move all the bits `N` positions to the right, subbing in **either** `N` `0`s or `N` (whatever the leftmost bit is)s on the left
  - Two different forms

```
1001 >> 2 =
either 0010 or 1110
```

Shift Right Trick

- Question: If shifting left multiplies, what does shift right do?

Shift Right Trick

- Question: If shifting left multiplies, what does shift right do?
    - Answer: divides in a similar way, but truncates result

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```
234 >> 1 =
23
```

Two Forms of Shift Right

- Subbing in `0`s makes sense
- What about subbing in the leftmost bit?
  - And why is this called "arithmetic" shift right?

```
1100 (arithmetic)>> 1 =
1110
```

Answer...Sort of

- Arithmetic form is intended for numbers in *twos complement*, whereas the non-arithmetic form is intended for *unsigned* numbers

Twos Complement

Problem

- Binary representation so far makes it easy to represent positive numbers and zero
- Question: What about representing negative numbers?

Twos Complement

- Way to represent positive integers, negative integers, and zero
- If `1` is in the *most significant bit* (generally leftmost bit in this class), then it is negative

Decimal to Twos Complement

- Example: -5 decimal to binary (twos complement)

Decimal to Twos Complement

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- First, convert the magnitude to an unsigned representation

Decimal to Twos Complement

- Example: -5 decimal to binary (twos complement)
- First, convert the magnitude to an unsigned representation

`5` (decimal) = `0101` (binary)

Decimal to Twos Complement

- Then, take the bits, and negate them

Decimal to Twos Complement

- Then, take the bits, and negate them

`0101`

Decimal to Twos Complement

- Then, take the bits, and negate them

```
~0101 =
 1010
```

Decimal to Twos Complement

- Finally, add one:

Decimal to Twos Complement

- Finally, add one:

```
1010
```

Decimal to Twos Complement

- Finally, add one:

```
1010 + 1 =
1011
```

Twos Complement to Decimal

- Same operation: negate the bits, and add one

Twos Complement to Decimal

- Same operation: negate the bits, and add one

```
1011
```

Twos Complement to Decimal

- Same operation: negate the bits, and add one

```
~1011 =
 0100
```

Twos Complement to Decimal

- Same operation: negate the bits, and add one

```
0100
```

Twos Complement to Decimal

- Same operation: negate the bits, and add one

```
0100 + 1 =
0101
```

Twos Complement to Decimal

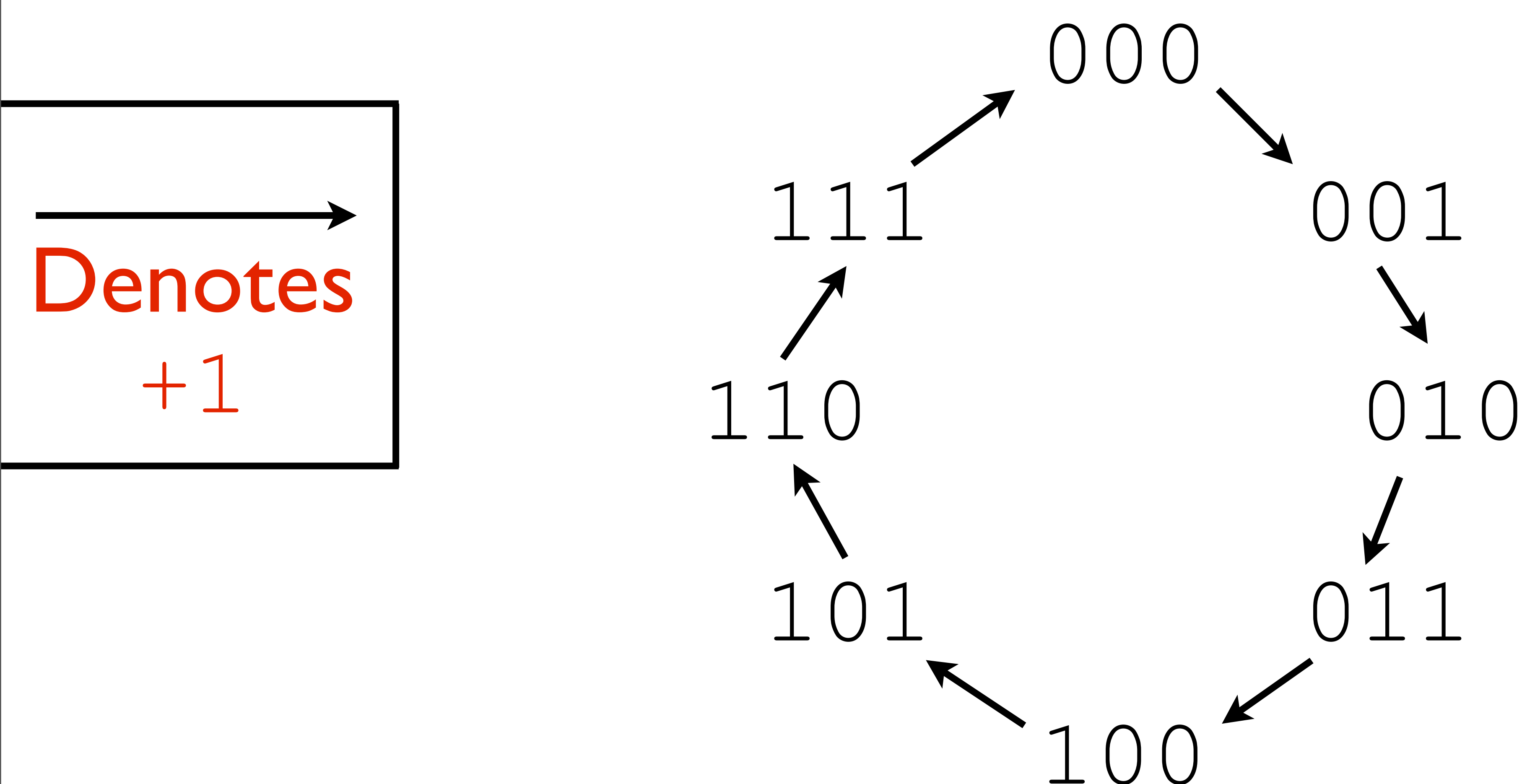
- Same operation: negate the bits, and add one

`0100 + 1 =`
`0101 =`
`-5`

We started with `1011` - negative

Intuition

- Modular arithmetic, with the convention that a leading `1` bit means negative



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Intuition

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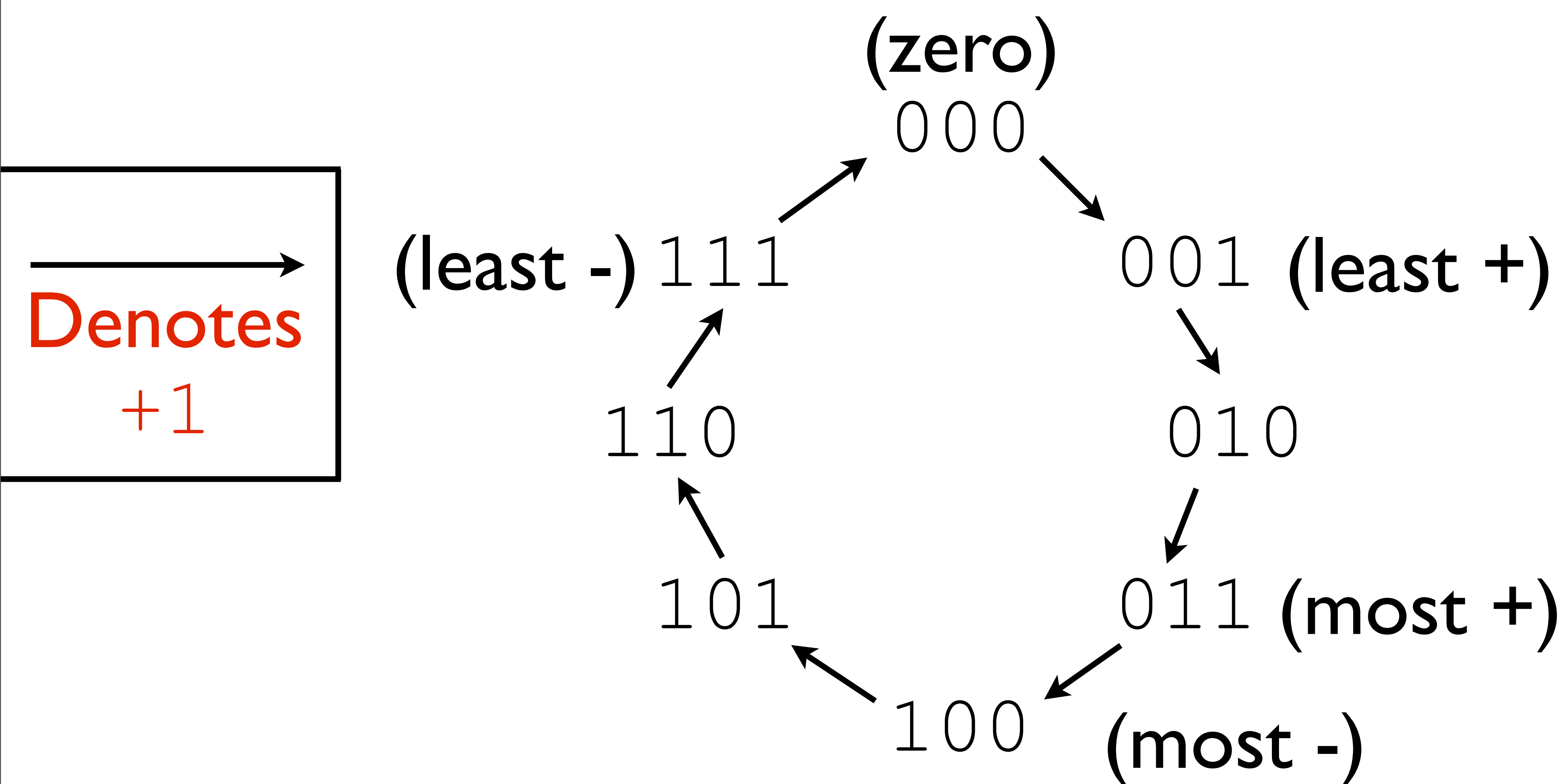

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Intuition

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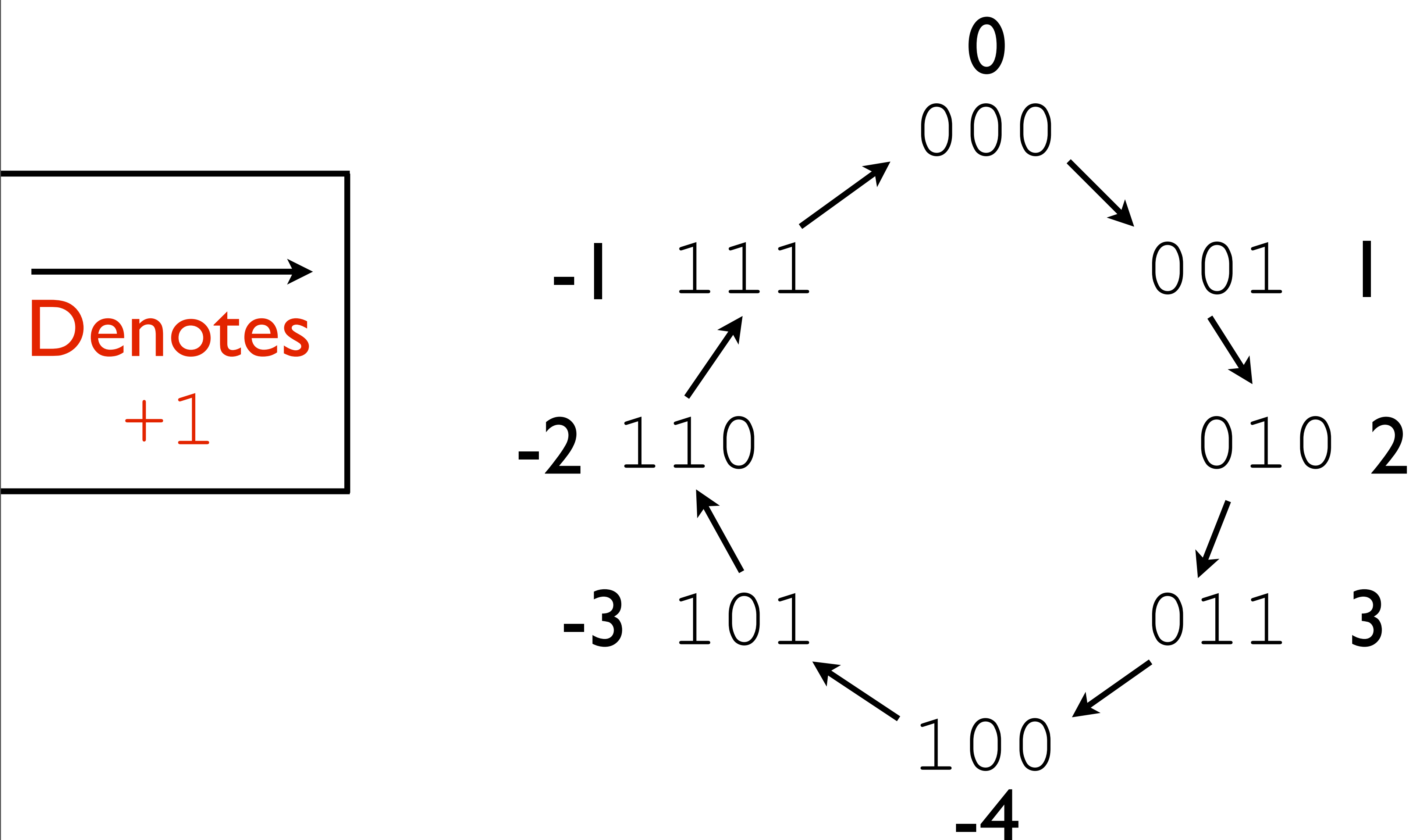

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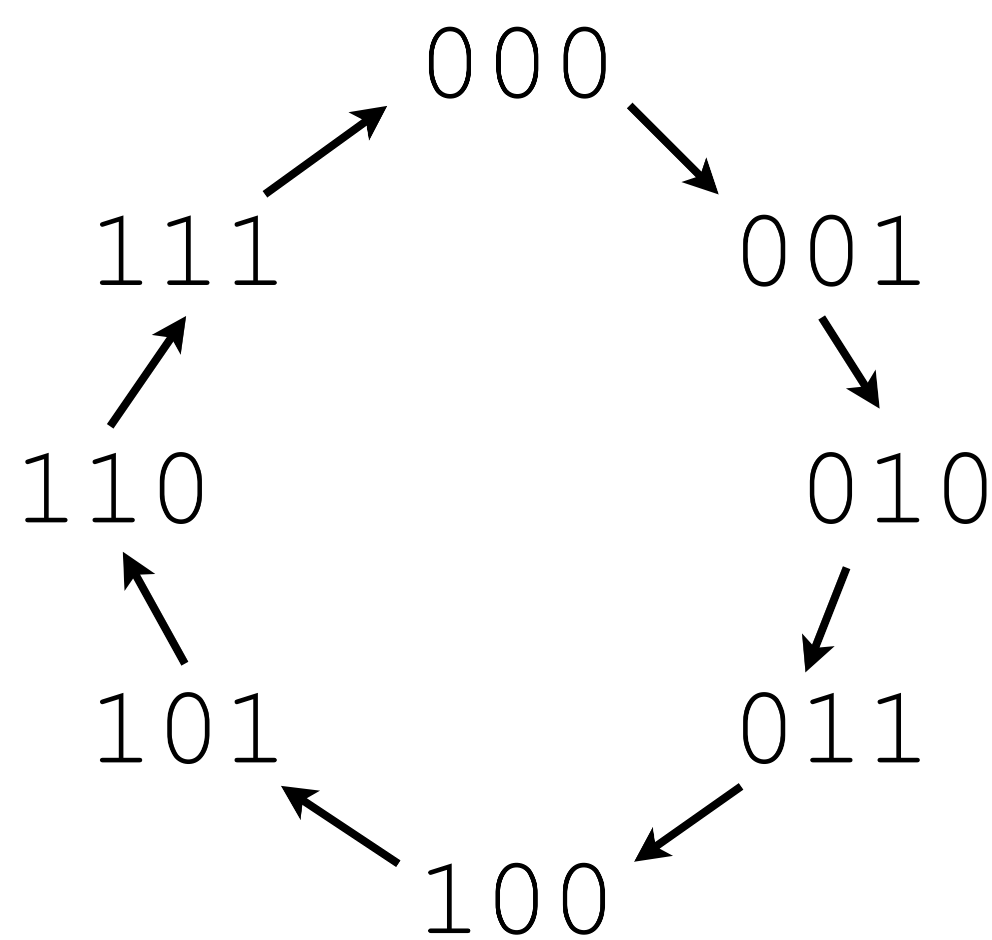
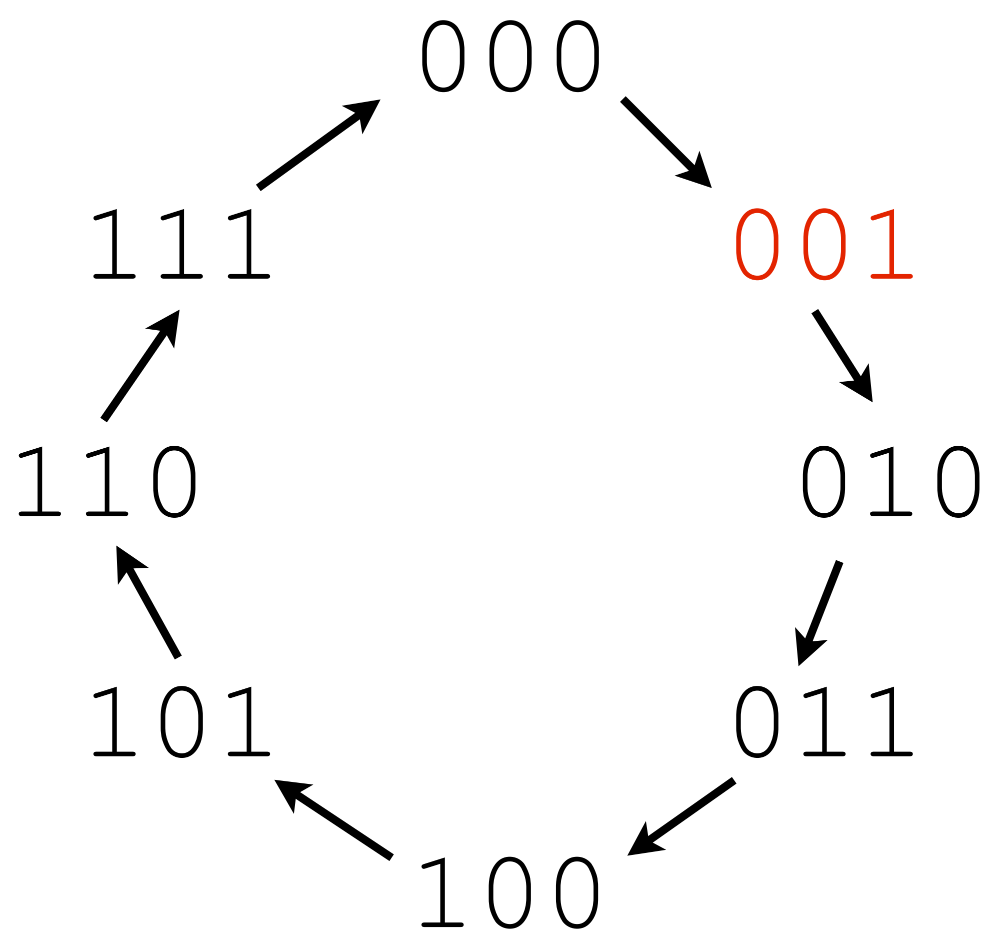
Negation of 1



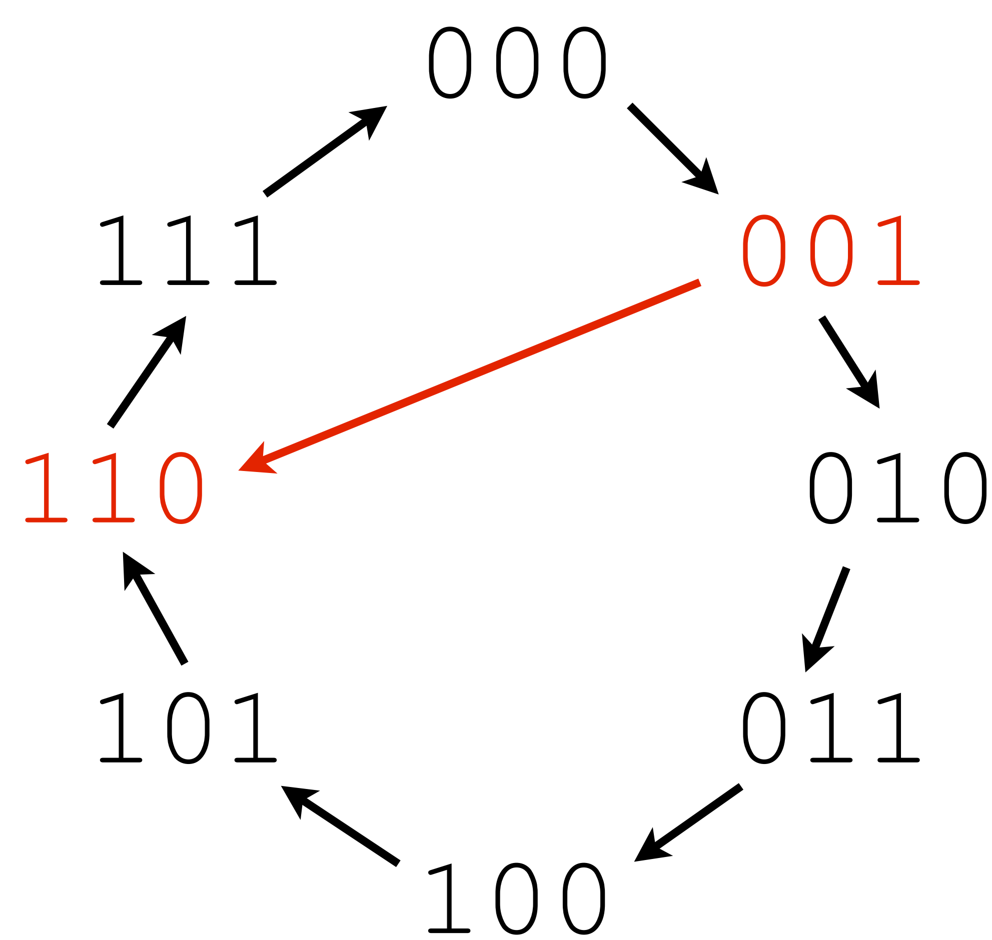
–Take our wheel from before

Negation of 1



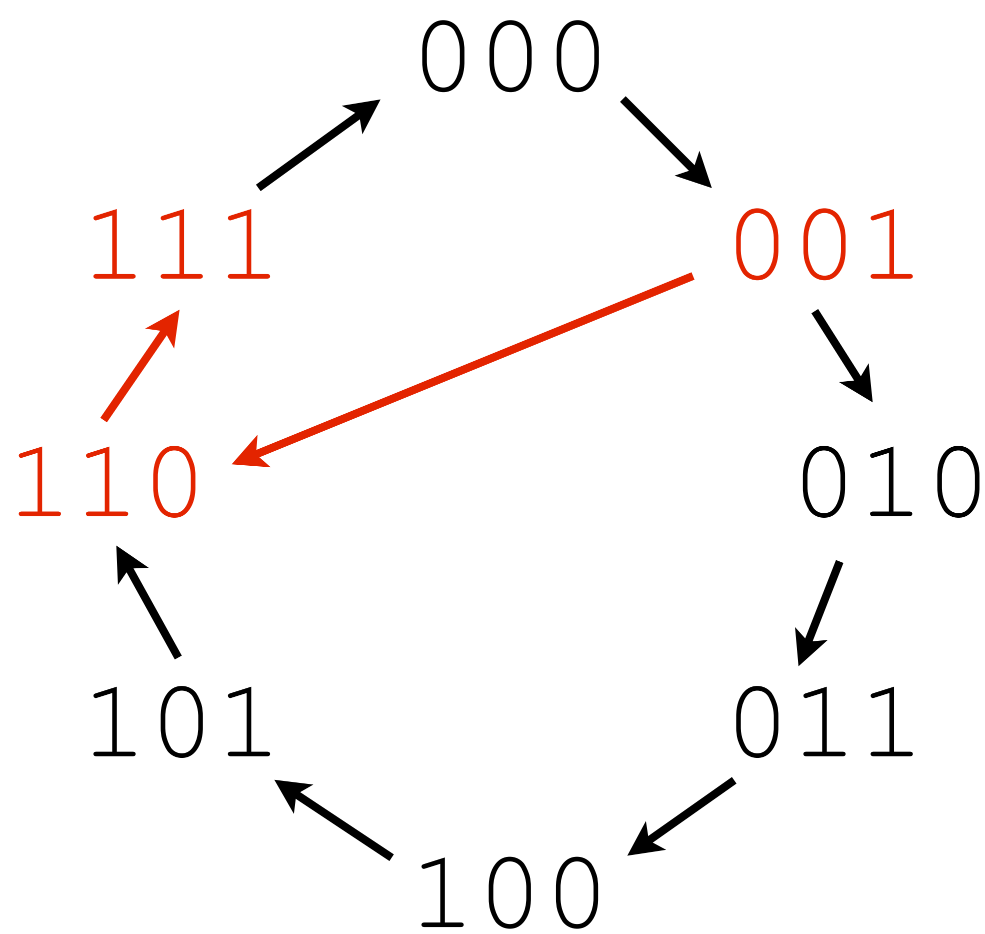
–This is 1

Negation of 1



–Inverted bits

Negation of 1

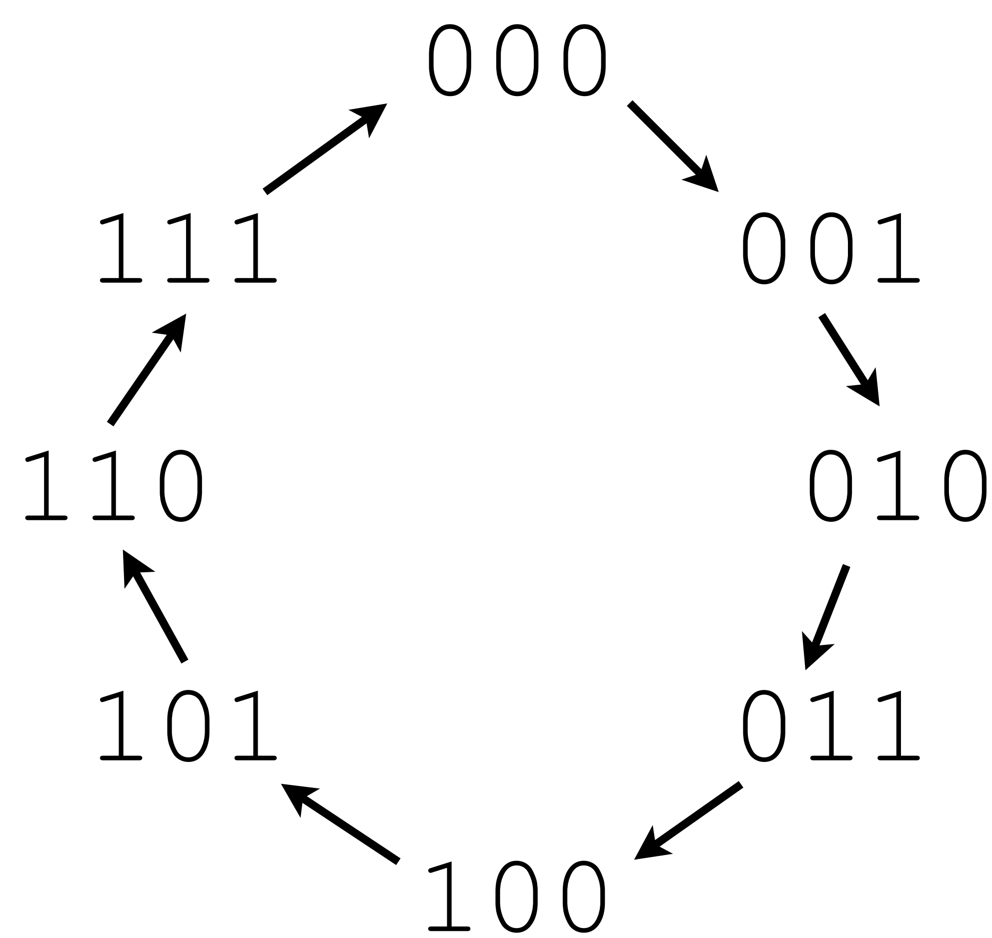


-Add 1

-This is exactly what we expected - binary 111 represents decimal -1

Consequences

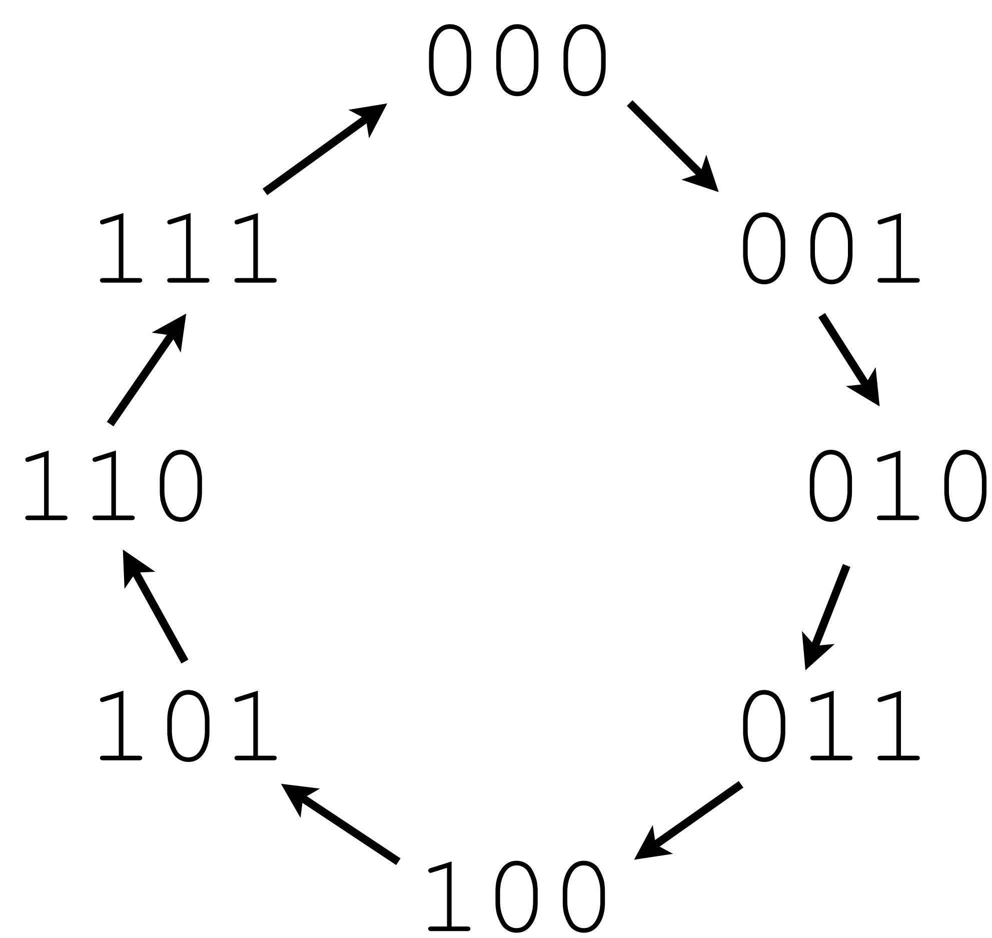
- What is the negation of `000`?



-Negate all bits: 000 -> 111
-Add one: 000
-Technically, adding one resulted in 1000, but that got cut off

Consequences

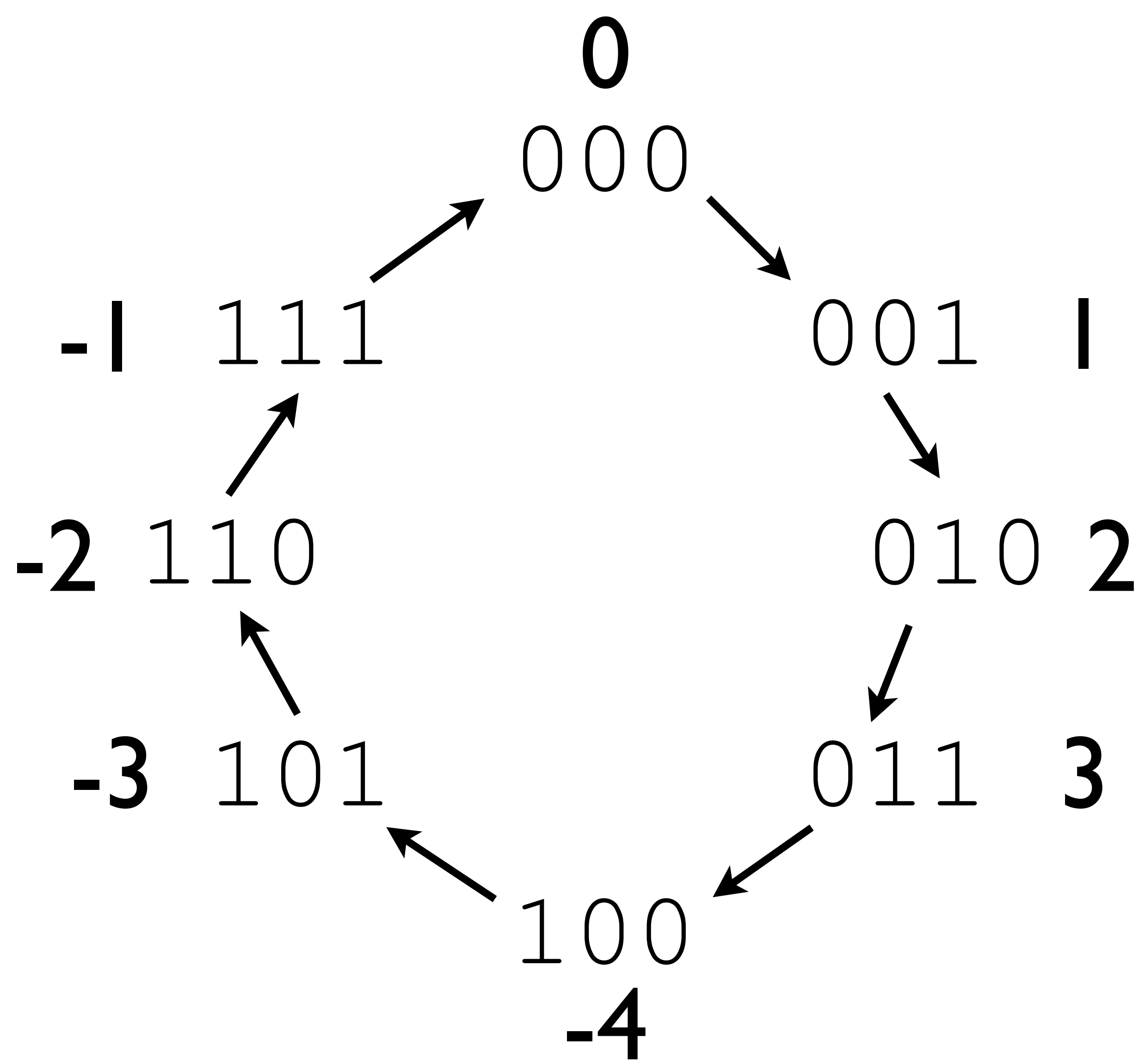
- What is the negation of `100`?



-Negate all bits: 100 -> 011
-Add one: 100
-Uh oh...this states that the negation of -4 is -4.
-Underlying problem is that we don't have a representation for 4 with just three bits

Arithmetic Shift Right

- **Not exactly** division by a power of two
- Consider -3 / 2



-101 (-3) shifted right yields 110 (-2), NOT 111 (-1) as expected from typical integer division
-Integer division rounds towards zero, whereas shift right rounds towards negative infinity
-This means they work _identically_ for positive values, but not for negative values (also meaning they are always the same for _unsigned_ values)

Addition

Building Up Addition

- Question: how might we add the following, in decimal?

```
  986
 +123
 ----
    ?
```

Building Up Addition

- Question: how might we add the following, in decimal?

```
 986
+123
----
   ?
```

			`6` `+3` `--` `?`

Building Up Addition

- Question: how might we add the following, in decimal?

```
 986
+123
----
   ?
```

		` 8` `+2` `--` ` ?`	` 6` `+3` `--` ` 9`

Building Up Addition

- Question: how might we add the following, in decimal?

```
 986
+123
----
   ?
```

	Carry: 1	8 +2 -- 0	6 +3 -- 9

Building Up Addition

- Question: how might we add the following, in decimal?

```
 986
+123
----
   ?
```

	``1 9 +1 -- ?```	```8 +2 -- 0```	```6 +3 -- 9```

Building Up Addition

- Question: how might we add the following, in decimal?

```
 986
+123
----
   ?
```

Carry: 1			
	1 9 +1 -- 1	8 +2 -- 0	6 +3 -- 9

Building Up Addition

- Question: how might we add the following, in decimal?

```
 986
+123
----
   ?
```

  1 +0 -- 1	1 9 +1 -- 1	8 +2 -- 0	6 +3 -- 9

Core Concepts

- We have a “primitive” notion of adding single digits, along with an idea of *carrying* digits
- We can build on this notion to add numbers together that are more than one digit long

Now in Binary

- Arguably simpler - fewer one-bit possibilities

<pre> 0 +0 -- ?</pre>	<pre> 0 +1 -- ?</pre>	<pre> 1 +0 -- ?</pre>	<pre> 1 +1 -- ?</pre>

Now in Binary

- Arguably simpler - fewer one-bit possibilities

`  0` `+0` `--` `  0`	`  0` `+1` `--` `  1`	`  1` `+0` `--` `  1`	`  1` `+1` `--` `  0` Carry: 1

Chaining the Carry

- Also need to account for any input carry

0 0 +0 -- 0	0 0 +1 -- 1	0 1 +0 -- 1	0 1 +1 -- 0 Carry: 1
1 0 +0 -- 1	1 0 +1 -- 0 Carry: 1	1 1 +0 -- 0 Carry: 1	1 1 +1 -- 1 Carry: 1

Adding Multiple Bits

- How might we add the numbers below?

```
 011
+001
------
```

Adding Multiple Bits

- How might we add the numbers below?

```
    0
  011
 +001
------
```

-Need an initial carry-in of zero

Adding Multiple Bits

- How might we add the numbers below?

```
   10
  011
 +001
------
    0
```

-Need an initial carry-in of zero

Adding Multiple Bits

- How might we add the numbers below?

```
  110
  011
 +001
------
   00
```

-Need an initial carry-in of zero

Adding Multiple Bits

- How might we add the numbers below?

```
 0110
  011
 +001
------
  100
```

-Need an initial carry-in of zero

Adding Multiple Bits

- How might we add the numbers below?

$$\begin{array}{r} \boxed{0}110 \\ 011 \\ +001 \\ \hline \boxed{100} \end{array}$$

Output Carry Bit

Result Bits

-Need an initial carry-in of zero

Another Example

```
  111
 +001
------
```

Another Example

```
   0
 111
+001
------
```

Another Example

```
  10
 111
+001
------
   0
```

Another Example

```
  110
  111
 +001
------
   00
```

Another Example

```
 1110
  111
 +001
------
  000
```

Output Carry Bit

Result Bits

-Now we have an output carry bit of 1. What does this mean?

Output Carry Bit Significance

- For unsigned numbers, it indicates if the result did not fit all the way into the number of bits allotted
- May be an error condition for software

Signed Addition

- Question: what is the result of the following operation?

```
 011
+011
----
   ?
```

Signed Addition

- Question: what is the result of the following operation?

```
 011
+011
----
0110
```

-If these are treated as signed numbers in two's complement, then we need a leading 0 to indicate that this is a positive number
-Truncated to three bits, the result is a negative number!

Overflow

- In this situation, *overflow* occurred: this means that both the operands had the same sign, and the result's sign differed

```
 011
+011
----
 110
```

- Possibly a software error

Overflow vs. Carry

- These are **different ideas**
  - Carry is relevant to **unsigned** values
  - Overflow is relevant to **signed** values

`111` `+001` `----` `000`	`011` `+011` `----` `110`	`111` `+100` `----` `011`	`001` `+001` `----` `010`
No Overflow; Carry	Overflow; No Carry	Overflow; Carry	No Overflow; No Carry

–As to when is it a problem, this all depends on exactly what it is you're doing

Subtraction

Subtraction

- Have been saying to invert bits and add one to second operand
- Could do it this way in hardware, but there is a trick

```
 001
-001
----
   ?
```

Hint: these two questions are equivalent

```
 001
+111
----
   ?
```

Subtraction Trick

- Assume we can cheaply invert bits, but we want to avoid adding twice (once to add 1 and once to add the other result)
- How can we do this easily?

Subtraction Trick

- Assume we can cheaply invert bits, but we want to avoid adding twice (once to add 1 and once to add the other result)
- How can we do this easily?
  - Set the initial carry to `1` instead of `0`

Subtraction Example

```
 0101
-0011
 ----
```

Subtraction Example

```
 0101
-0011
 ----
```

Invert `0011` $\longrightarrow$

Subtraction Example

```
 0101
-0011
 ----
```

Invert `0011` → `1100`

Subtraction Example

```
 0101
-0011
 ----
```

Invert `0011` → `1100`

Equivalent to →

Subtraction Example

```
  0101      Invert 0011          Equivalent to       1
 -0011      ----------->  1100  --------------->   0101
  ----                                            +1100
                                                   ----
```

-An initial carry-in of 1 is equivalent to adding 1 and then adding the other operand

Subtraction Example

0101
−0011

Invert 0011 $\longrightarrow$ 1100 Equivalent to $\longrightarrow$

$\boxed{1}1011$
0101
+1100

0010

–An initial carry–in of 1 is equivalent to adding 1 and then adding the other operand