

COMP 122/L Lecture 3

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Outline

- Operations on binary values
 - Addition
 - Subtraction
- Floating point introduction

Addition

Building Up Addition

- Question: how might we add the following, in decimal?

$$\begin{array}{r} 986 \\ +123 \\ \hline \end{array}$$

?

Building Up Addition

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			$\begin{array}{r} 6 \\ +3 \\ \hline \end{array}$ <p>?</p>
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Building Up Addition

- Question: how might we add the following, in decimal?

$$\begin{array}{r} 986 \\ +123 \\ \hline \end{array}$$

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$$\begin{array}{r} 8 \\ +2 \\ \hline \end{array}$$

?

$$\begin{array}{r} 6 \\ +3 \\ \hline \end{array}$$

9

Building Up Addition

- Question: how might we add the following, in decimal?

$$\begin{array}{r} 986 \\ +123 \\ \hline \end{array}$$

?

Carry: 1

$$\begin{array}{r} 8 \\ +2 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 6 \\ +3 \\ \hline 9 \end{array}$$

Building Up Addition

- Question: how might we add the following, in decimal?

$$\begin{array}{r} 986 \\ +123 \\ \hline \end{array}$$

?

	$\begin{array}{r} 1 \\ 9 \\ +1 \\ \hline \end{array}$ <p>?</p>		
--	--	--	--

	$\begin{array}{r} 8 \\ +2 \\ \hline \end{array}$ <p>0</p>		
--	---	--	--

	$\begin{array}{r} 6 \\ +3 \\ \hline \end{array}$ <p>9</p>		
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Building Up Addition

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Building Up Addition

- Question: how might we add the following, in decimal?

$$\begin{array}{r} 986 \\ +123 \\ \hline \end{array}$$

?

$$\begin{array}{r} 1 \\ +0 \\ \hline 1 \end{array}$$

$$\begin{array}{r} 1 \\ 9 \\ +1 \\ \hline 1 \end{array}$$

$$\begin{array}{r} 8 \\ +2 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 6 \\ +3 \\ \hline 9 \end{array}$$

Core Concepts

- We have a “primitive” notion of adding single digits, along with an idea of *carrying* digits
- We can build on this notion to add numbers together that are more than one digit long

Now in Binary

- Arguably simpler - fewer one-bit possibilities

0	0	1	1
+0	+1	+0	+1
--	--	--	--
?	?	?	?

Now in Binary

- Arguably simpler - fewer one-bit possibilities

0
+0
--
0

0
+1
--
1

1
+0
--
1

1
+1
--
0

Carry: 1

Chaining the Carry

- Also need to account for any input carry

<div>0</div> <div>0</div> <div>+0</div> <div>--</div> <div>0</div>	<div>0</div> <div>0</div> <div>+1</div> <div>--</div> <div>1</div>	<div>0</div> <div>1</div> <div>+0</div> <div>--</div> <div>1</div>	<div>0</div> <div>1</div> <div>+1</div> <div>--</div> <div>0</div> <div>Carry: 1</div>
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Adding Multiple Bits

- How might we add the numbers below?

$$\begin{array}{r} 011 \\ +001 \\ \hline \end{array}$$

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Adding Multiple Bits

- How might we add the numbers below?

$$\begin{array}{r} 10 \\ 011 \\ +001 \\ \hline 0 \end{array}$$

Adding Multiple Bits

- How might we add the numbers below?

$$\begin{array}{r} 110 \\ 011 \\ +001 \\ \hline 00 \end{array}$$

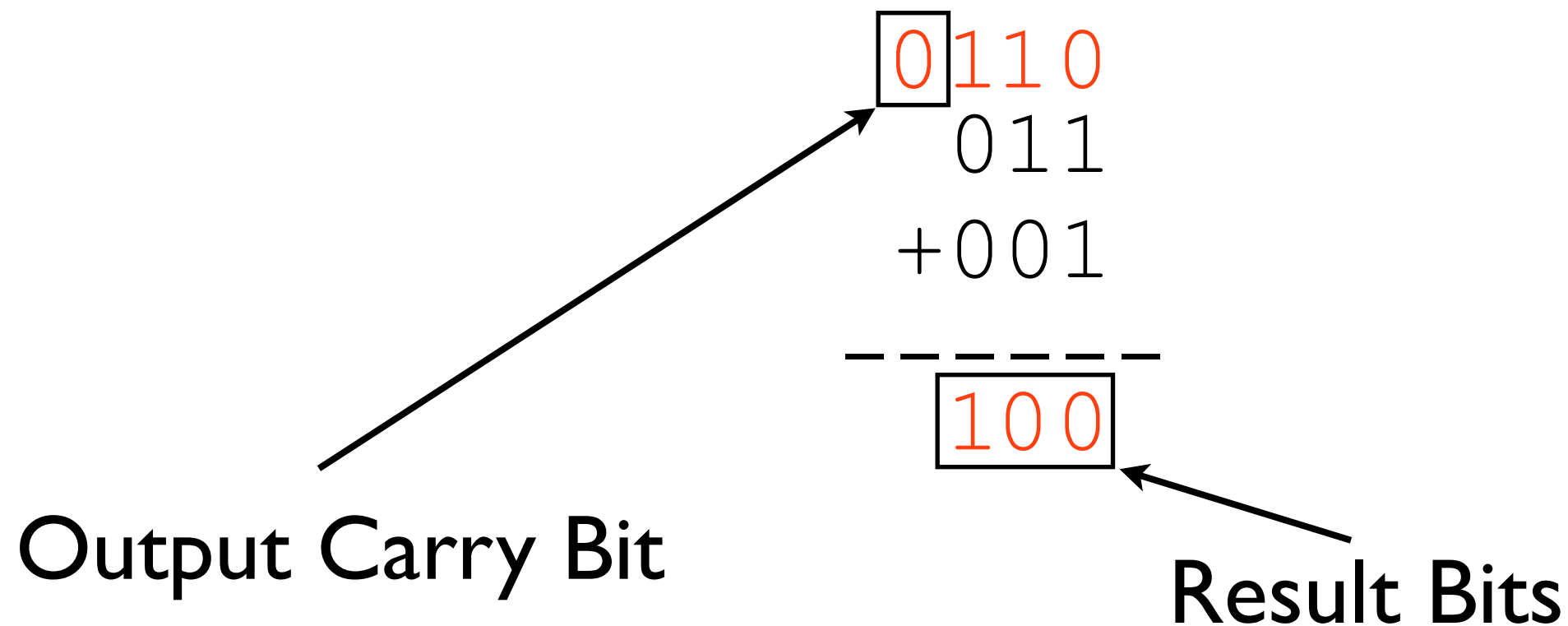
Adding Multiple Bits

- How might we add the numbers below?

$$\begin{array}{r} 0110 \\ 011 \\ +001 \\ \hline 100 \end{array}$$

Adding Multiple Bits

- How might we add the numbers below?



Another Example

$$\begin{array}{r} 111 \\ +001 \\ \hline \end{array}$$

Another Example

$$\begin{array}{r} 0 \\ 111 \\ +001 \\ \hline \end{array}$$

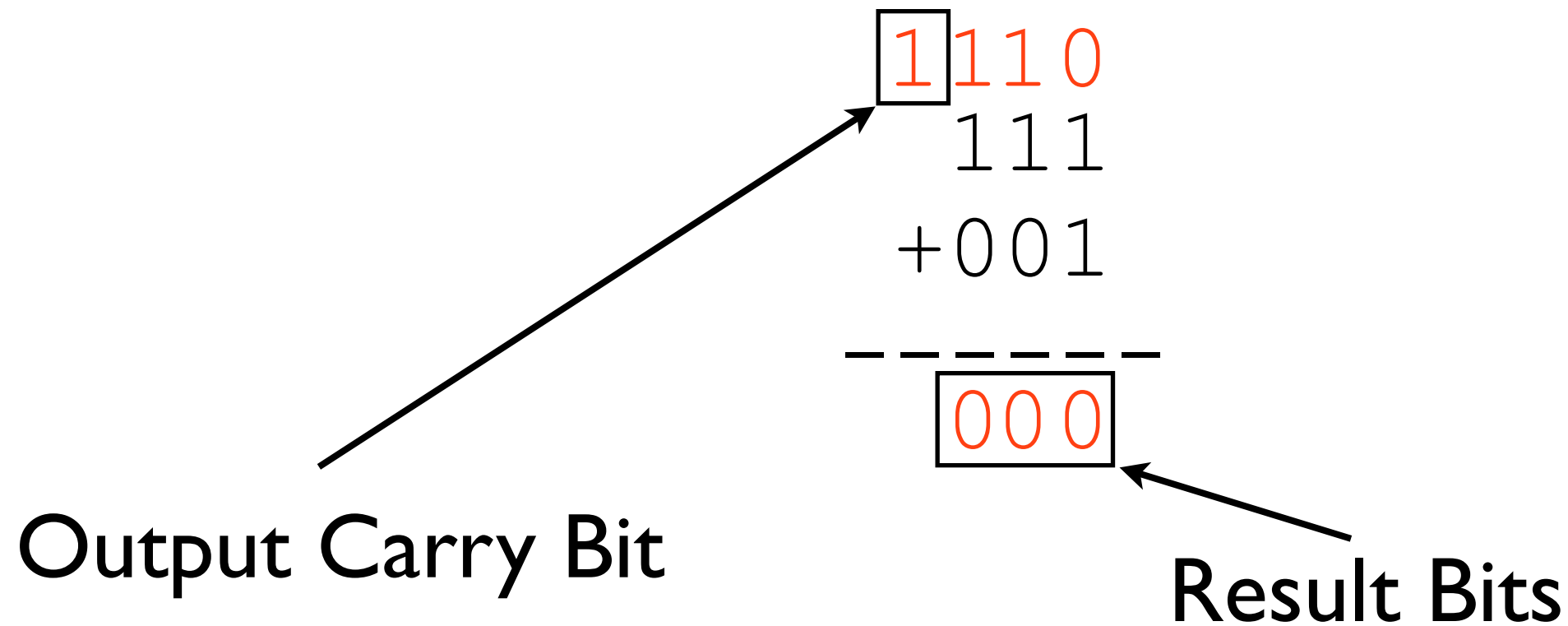
Another Example

$$\begin{array}{r} 10 \\ 111 \\ +001 \\ \hline 0 \end{array}$$

Another Example

$$\begin{array}{r} 110 \\ 111 \\ +001 \\ \hline 00 \end{array}$$

Another Example



Output Carry Bit Significance

- For unsigned numbers, it indicates if the result did not fit all the way into the number of bits allotted
- May be an error condition for software

Signed Addition

- Question: what is the result of the following operation?

$$\begin{array}{r} 011 \\ +011 \\ \hline \end{array}$$

?

Signed Addition

- Question: what is the result of the following operation?

$$\begin{array}{r} 011 \\ +011 \\ \hline 0110 \end{array}$$

Overflow

- In this situation, *overflow* occurred: this means that both the operands had the same sign, and the result's sign differed

$$\begin{array}{r} 011 \\ +011 \\ \hline 110 \end{array}$$

- Possibly a software error

Overflow vs. Carry

- These are **different ideas**
 - Carry is relevant to **unsigned** values
 - Overflow is relevant to **signed** values

111
+001

000

No Overflow;
Carry

011
+011

110

Overflow;
No Carry

111
+100

011

Overflow;
Carry

001
+001

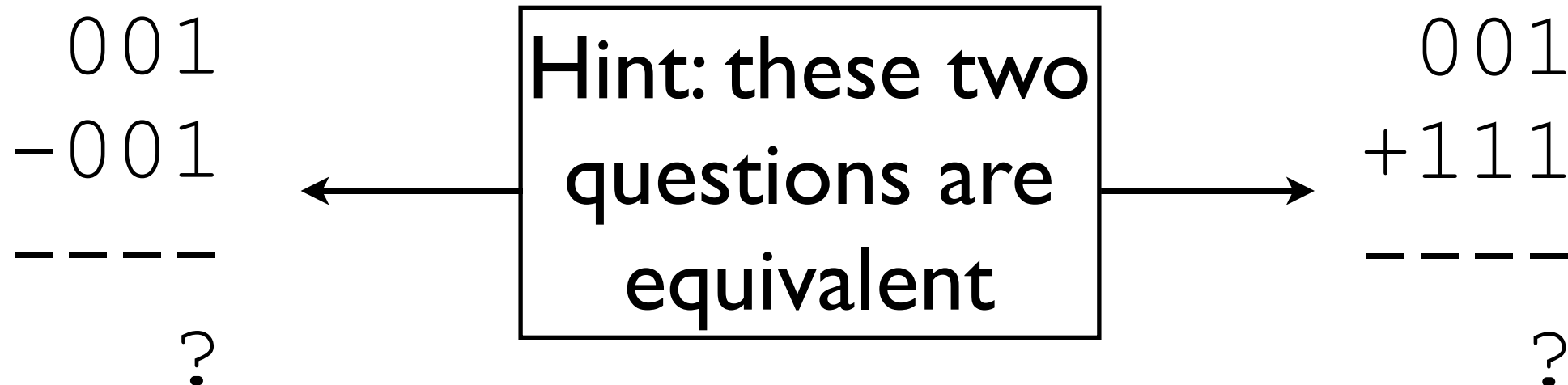
010

No Overflow;
No Carry

Subtraction

Subtraction

- Have been saying to invert bits and add one to second operand
- Could do it this way in hardware, but there is a trick



Subtraction Trick

- Assume we can cheaply invert bits, but we want to avoid adding twice (once to add 1 and once to add the other result)
- How can we do this easily?

Subtraction Trick

- Assume we can cheaply invert bits, but we want to avoid adding twice (once to add 1 and once to add the other result)
- How can we do this easily?
 - Set the initial carry to 1 instead of 0

Subtraction Example

$$\begin{array}{r} 0101 \\ -0011 \\ \hline \end{array}$$

Subtraction Example

$$\begin{array}{r} 0101 \\ -0011 \\ \hline \end{array} \quad \begin{array}{l} \text{Invert } 0011 \\ \hline \end{array} \rightarrow$$

Subtraction Example

$$\begin{array}{r} 0101 \\ -0011 \\ \hline \end{array} \quad \begin{array}{c} \text{Invert } 0011 \\ \hline \end{array} \quad 1100$$

Subtraction Example

0101
-0011

Invert 0011



1100

Equivalent to



Subtraction Example

$$\begin{array}{r} 0101 \\ -0011 \\ \hline \end{array} \xrightarrow{\text{Invert } 0011} 1100 \xrightarrow{\text{Equivalent to}} \begin{array}{r} 0101 \\ +1100 \\ \hline \end{array}$$

The diagram illustrates the subtraction of 0011 from 0101 using the two's complement method. The first step shows the subtraction problem. The second step shows the result of inverting the subtrahend (0011) to 1100. The third step shows the equivalent addition problem, where the minuend (0101) is added to the inverted subtrahend (1100). A red '1' is placed above the final result line, indicating a carry-out or overflow.

Subtraction Example

$$\begin{array}{r} 0101 \\ -0011 \\ \hline \end{array}$$

Invert 0011



$$1100$$

Equivalent to



$$\begin{array}{r} \boxed{1}1011 \\ 0101 \\ +1100 \\ \hline 0010 \end{array}$$

Floating Point Introduction

Question

How might we represent floating point numbers?

1 . 2 5

4 7 . 9

0 . 8 2

Enter IEEE-754

- Standardized floating point representation and operations
- Modern systems all use this
- Complex and *weird*

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`min(X, Y) ==? min(Y, X)`

Enter IEEE-754

- Standardized floating point representation and operations
- Modern systems all use this
- Complex and *weird*

`min(X, Y) ==? min(Y, X)`

May or may not be true...

Basis

Based on the idea of *scientific notation*

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$$4.23 * 10^7$$

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Save these

Basis

Based on the idea of *scientific notation*

$$4.23 * 10^7$$

Save these

Caveat: this is in binary

$$1.1 * 2^{-1}$$

Components

$$1.1 * 2^{-1}$$

- Sign bit (+/-)
- Exponent
- Fraction / mantissa