COMP 122/L Lecture 3

Kyle Dewey

Outline

- Floating point numbers

Question

How might we represent floating point numbers?

```
1.25
47.9
0.82
```

-A lot of different ways possible
-A whole lot of problems related to precision arise. Just about any representation devisable will be complex.

Enter IEEE-754

- Standardized floating point representation and operations
- Modern systems all use this
- Complex and *weird*

Enter IEEE-754

- Standardized floating point representation and operations
- Modern systems all use this
- Complex and *weird*

```
min(X, Y) =? min(Y, X)
```

Enter IEEE-754

- Standardized floating point representation and operations
- Modern systems all use this
- Complex and *weird*

`min(X, Y) =? min(Y, X)`

May or may not be true...

–Standard doesn't enforce that this is true in general. Implementations are permitted to make it so this isn't true in all cases.

Basis

Based on the idea of *scientific notation*

Basis

Based on the idea of *scientific notation*

$4.23 * 10^7$ $-8.7 * 10^2$ $14.6 * 10^{-5}$ $-9.4 * 10^{-18}$

Basis

Based on the idea of *scientific notation*

$4.23 * 10^{7}$ $-8.7 * 10^{2}$ $14.6 * 10^{-5}$ $-9.4 * 10^{-18}$

Caveat: use powers of two

$1.1 * 2^{-1}$

Basis

Based on the idea of *scientific notation*

$4.23 * 10^{7}$ $-8.7 * 10^{2}$ $14.6 * 10^{-5}$ $-9.4 * 10^{-18}$

Caveat: use powers of two

$1.1 * 2^{-1}$

Additional caveat: numbers are always in the form:
$+/- 1.XXX * 2^{YYYY}$

Components

$$\boxed{+/-}\ 1.\boxed{XXX} * 2^{\boxed{YYYY}}$$

sign bit

fraction
(AKA mantissa)

exponent

–Image from: https://en.wikipedia.org/wiki/IEEE_754

Components

$$\boxed{+/-}\ 1.\boxed{XXX} * 2^{\boxed{YYYY}}$$

sign bit

fraction
(AKA mantissa)

exponent

- More bits for fraction and exponent mean greater range and precision
- Most common: 32 bits (binary32 / `float`) and 64 bits (binary64 / `double`)

–Image from: https://en.wikipedia.org/wiki/IEEE_754

Components

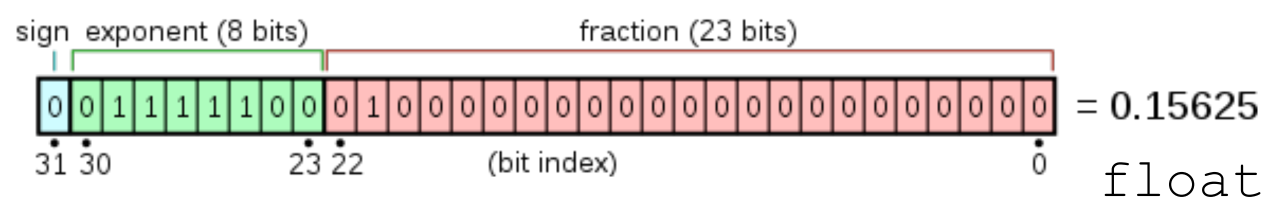
$\boxed{+/-}\ 1.\boxed{XXX} * 2^{\boxed{YYYY}}$

sign bit

fraction
(AKA mantissa)

exponent

- More bits for fraction and exponent mean greater range and precision
- Most common: 32 bits (binary32 / `float`) and 64 bits (binary64 / `double`)



`float`

–Image from: https://en.wikipedia.org/wiki/IEEE_754

Question

$$\boxed{+/-}\ 1.\boxed{XXX} * 2^{\boxed{YYYY}}$$

sign bit

fraction
(AKA mantissa)

exponent

Standard says:

Sign Bit	Exponent	Fraction

Why this order?

Question

$$\boxed{+/-}\ 1.\boxed{XXX} * 2^{\boxed{YYYY}}$$

sign bit

fraction
(AKA mantissa)

exponent

Standard says:

Sign Bit	Exponent	Fraction

Why this order?

(Usually) preserves order even if compared as a two's complement integer

Sign Bit

0 = positive; 1 = negative

Question: What about 0?

Sign Bit

0 = positive; 1 = negative

Question: What about 0?

Both positive and negative zero (quirk).

Fraction Value

Recall: each bit for integers represents a power of two:

1001

1	0	0	1	
1 * 2^3	0 * 2^2	0 * 2^1	1 * 2^0	
8	0	0	1	= 9

Same idea for fractional part, but with *negative exponents:*

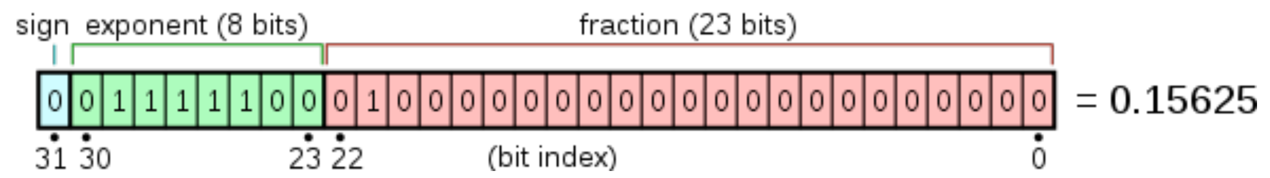
XXXX.0110

0	1	1	0	
0 * 2^-1	1 * 2^-2	1 * 2^-3	0 * 2^-4	
0	0.25	0.125	0	= 0.375

Exponent Value

- Always written as an unsigned number, *even for negative exponents*
- Uses a *biased representation*: the actual exponent is always (written exponent - 127)
    - If written exponent is 120, the actual exponent is (120 - 127) = -7

Putting it All Together



- Sign: 0 (positive)
- Exponent: 4 + 8 + 16 + 32 + 64 = 124; 124 - 127 = -3
- Fraction: $0 * 2^{-1} + 1 * 2^{-2} = 0.25$
- Overall magnitude: $(1 + 0.25) * 2^{-3} = 0.15625$

The 1 is implicit in the encoding

$+/- \ 1.XXX * 2^{YYYY}$

Decimal Floating-point to Binary Floating-point

Floating-point Conversion

- Basic idea: determine the correct sign bit, exponent, and fractional part to use, and stitch them together
- Eight-step algorithm for this
- Running example: -9.5625

Step 1: Determine Sign Bit

- -9.5625 is negative
- Sign bit is 1 for negative values

Step 2: Convert Integral Part to Unsigned Binary

- -9.5625's integral portion is 9
- 9 = 1001
- No need to add padding or anything else (yet)

Step 3: Convert Fractional Part to Binary

- -9.5625's fractional portion is 0.5625
- Determine which negative powers of two (2^{-1}, 2^{-2}, 2^{-3}, ...) will sum up to this number, or at least as closely as possible to this number
- Pseudocode algorithm on next side can be used for this

```
fraction = 0.5625
num_iterations = 0
bits = ""
while fraction != 0 and
      num_iterations < 23:
  fraction *= 2
  num_iterations++
  if fraction >= 1.0:
    bits += "1"
    fraction -= 1.0
  else:
    bits += "0"
```

Step 3: Algorithm with Example

0.5625

Iteration	Calculation	>= 1.0?	Output Bit
1	0.5625 * 2 = 1.125	yes	1
2	0.125 * 2 = 0.25	no	0
3	0.25 * 2 = 0.5	no	0
4	0.5 * 2 = 1.0	yes	1

Step 4: Normalize Value

- Recall: the encoding assumes numbers are always in the format +/- 1.XXX * 2^{YYYY}
- We need to put the number in this form
- Integral part (step 2): 1001
- Fractional part (step 3): 1001
- Number overall: integral.fractional: 1001.1001

Step 4: Normalize Value

- To get 1001.1001 into the expected 1.XXX * 2^{YYYY} format, we need to move the dot to the left 3 positions
- Moves to the left denote positive exponents, moves to the right are negative
- Overall: exponent: 3

Step 5: Add Bias to Exponent

- Exponent (from step 4): 3
- Recall: exponents are stored in biased form, and we will always subtract 127 from this value **later**
- ...so here, we always **add** 127: 3 + 127 = 130

Step 6: Convert Biased Exponent to Binary

- Biased exponent (step 5): 130
- 130 in binary: 1000 0010

Step 7: Determine Final Mantissa Bits

- Needed: exactly 23 mantissa bits
- From step 4, we initially had 1001.1001, then moved the dot to the left to get 1.0011001
- First bits of mantissa here will thus be 0011001

Step 7: Remaining Mantissa Bits

- Initial bits: 0011001
- If algorithm in step 3 terminated with 1.0, the number is getting represented exactly, with no precision loss. Pad zeros on right until 23 bits.
  - May need additional algorithm iterations if some precision loss happened
- Depending on how exponent moved, some bits on the right may also need to be removed (precision loss)

Step 7: Remaining Mantissa Bits

- Initial bits: 0011001
- Algorithm terminated exactly, so we can pad with zeros. Final mantissa:
  - 0011 0010 0000 0000 0000 000

Step 8: Combine Everything

- Sign bit (step 1): 1
- Exponent bits (step 6): 1000 0010
- Mantissa bits (step 7): 0011 0010 0000 0000 0000 000
- Overall: 1 1000 0010 0011 0010 0000 0000 0000 000 (copy/pasting all components)
  - Or: 1100 0001 0001 1001 0000 0000 0000 0000 (spaces every nibble)

Further Examples / Explanation

- There is also a link named "Instructions for Converting Between Decimal and Binary Floating-Point Numbers" linked off of the course website
  - Covers the same thing, but says it a little differently, and has additional examples and links