

COMP 410
Fall 2020

Purely Logical Arithmetic with Peano Axioms

For these problems, we'll define natural numbers via the Peano axioms, where n is a natural number if it is `zero`, or if n is of the form `succ(m)`, where m is another natural number.

1.) Write out 3 as a natural number encoded with the Peano axioms.

2.) Define a procedure named `add`, which adds two natural numbers together. As a hint, $0 + n = n$, and $\text{succ}(n) + m = \text{succ}(n + m)$. Example queries are below:

```
?- add(succ(zero), succ(zero), Result). % 1 + 1 = ???  
Result = succ(succ(zero)). % 2
```

```
?- add(succ(succ(zero)), Other, succ(succ(succ(zero)))). % 2 + ??? = 3  
Other = succ(zero). % 1
```

3.) Define a procedure named `lt`, which succeeds if the first argument is less than the second. As a hint, $0 < n$, where $n > 0$. Additionally, $(n + 1) < (m + 1)$ if $n < m$. Example queries are below:

```
?- lt(succ(zero), succ(succ(zero))). % 1 < 2  
true.
```

```
?- lt(Something, succ(succ(zero))). % ??? < 2  
Something = zero ;  
Something = succ(zero) ;  
false.
```

4.) Define a procedure named `lte`, which succeeds if the first argument is \leq the second. As a hint, $n \leq n$ for all natural numbers n , and $n \leq m$ if $n < m$.

5.) Reusing the procedures written above, write a query to find all natural numbers N and M such that $N + M = 5$, and $N \leq M$.