COMP 410 Lecture 2

Kyle Dewey

SAT and Semantic Tableau

SAT Background

SAT

- Short for the Boolean satisfiability problem
- Given a Boolean formula with variables, is there an assignment of true/false to the variables which makes the formula true?

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$(x \land \neg x)$

No

Relevance

Widespread usage in hardware and software verification

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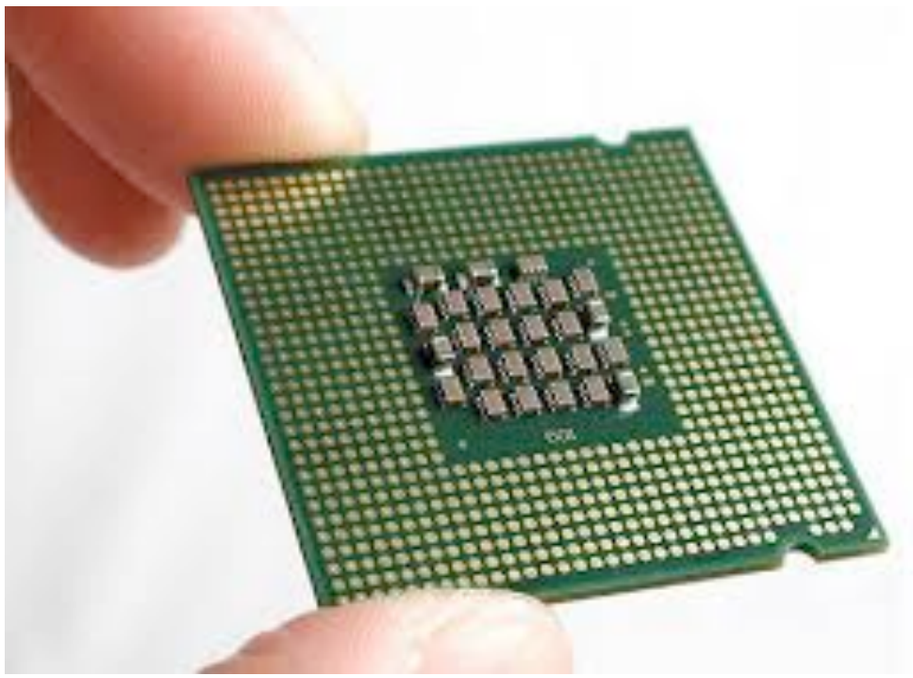
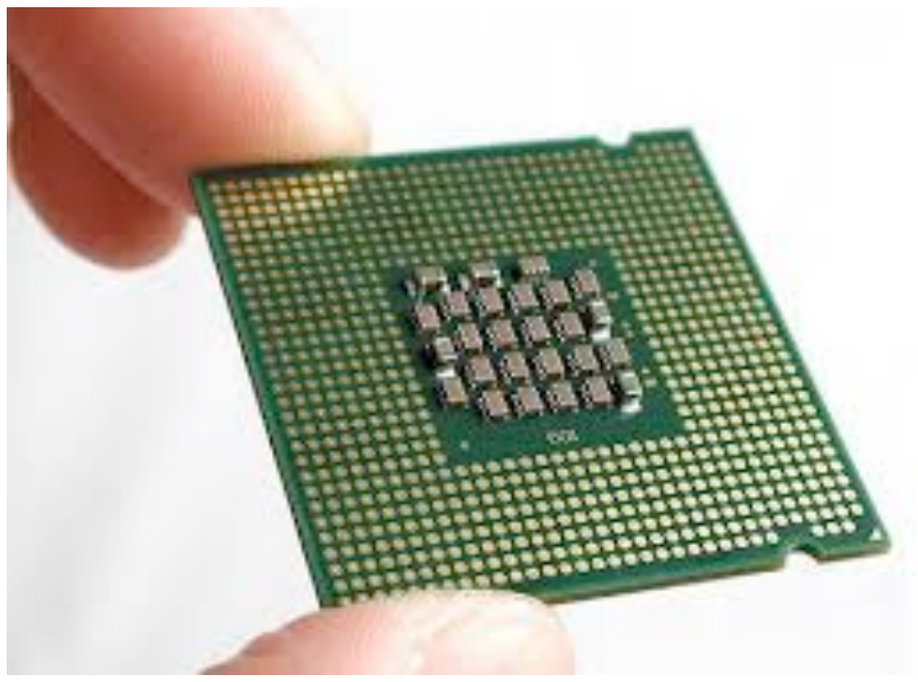


Relevance

Widespread usage in hardware and software verification



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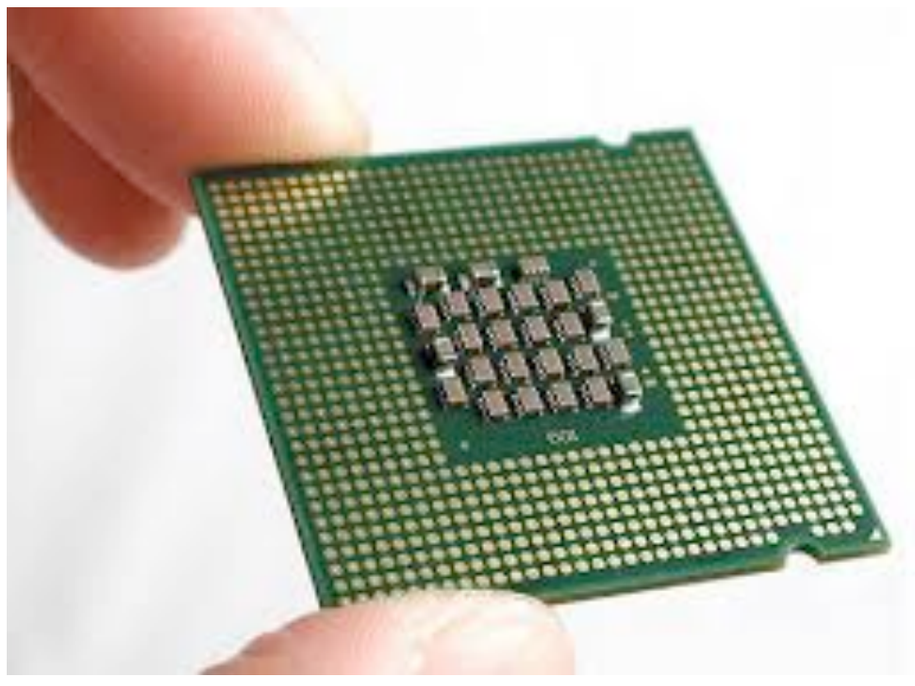


Relevance to Logic Programming

- Methods for solving SAT can be used to execute logic programs
- Logic programming can be phrased as SAT with some additional stuff

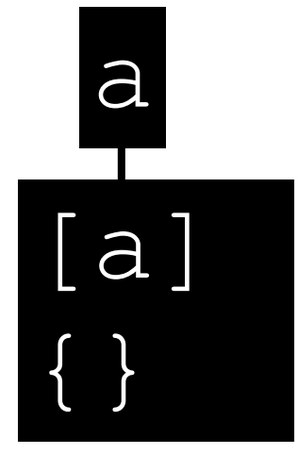
Semantic Tableau

- One method for solving SAT instances
- Basic idea: iterate over the formula
  - Maintain subformulas that must be true
  - Learn which variables must be true/false
  - Stop at conflicts (unsatisfiable), or when no subformulas remain (have solution)

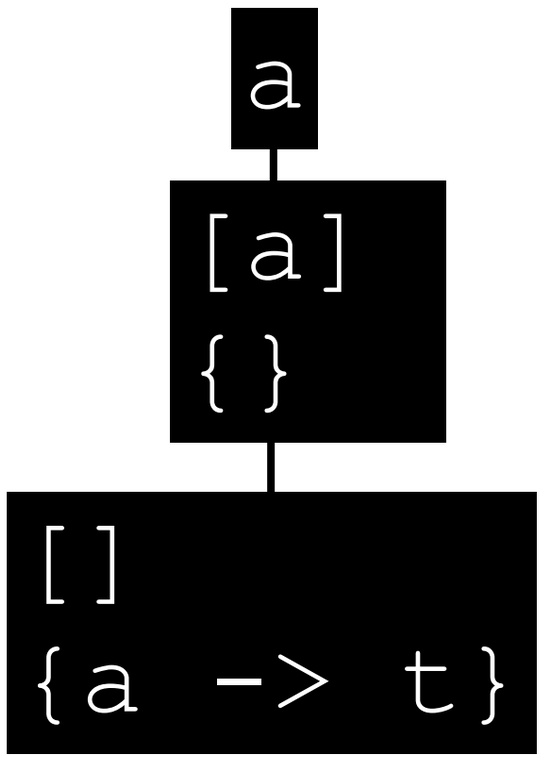
Positive Literals

a

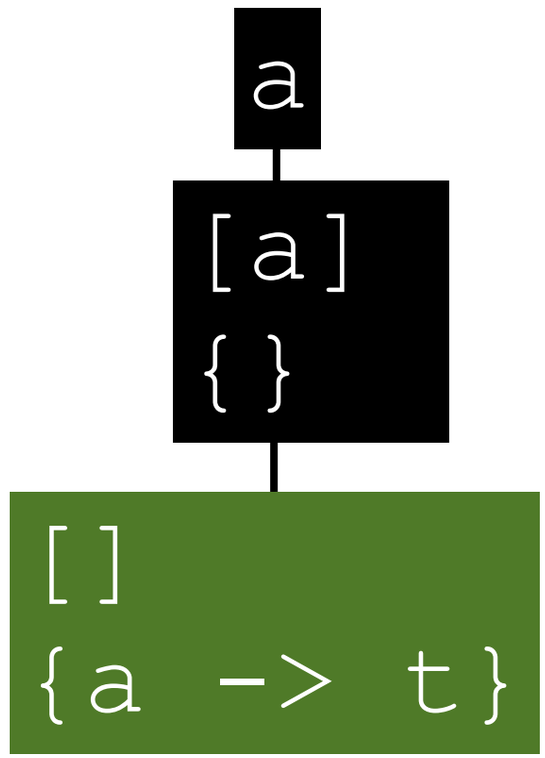
Positive Literals



Positive Literals



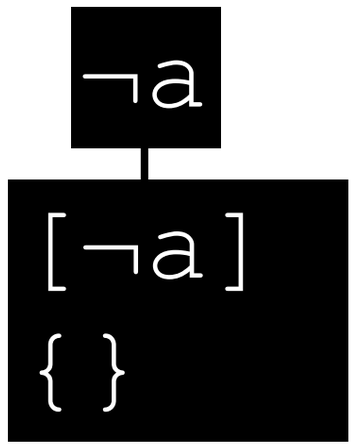
Positive Literals



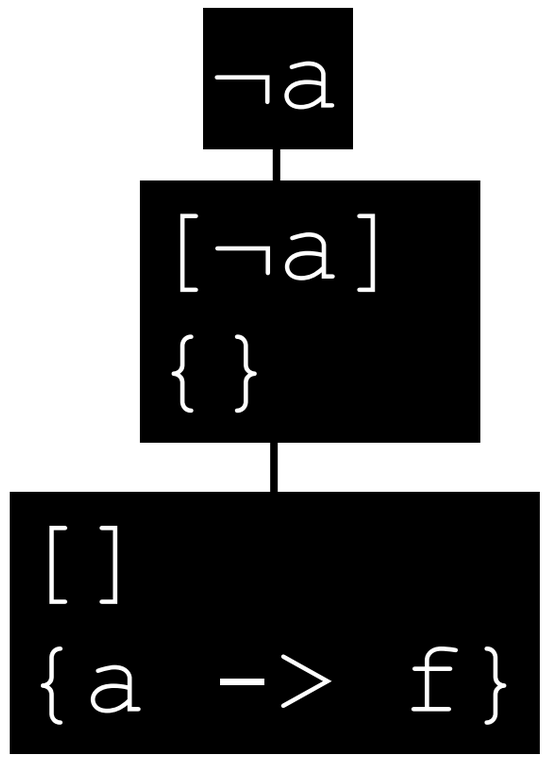
Negative Literals

$\neg a$

Negative Literals



Negative Literals



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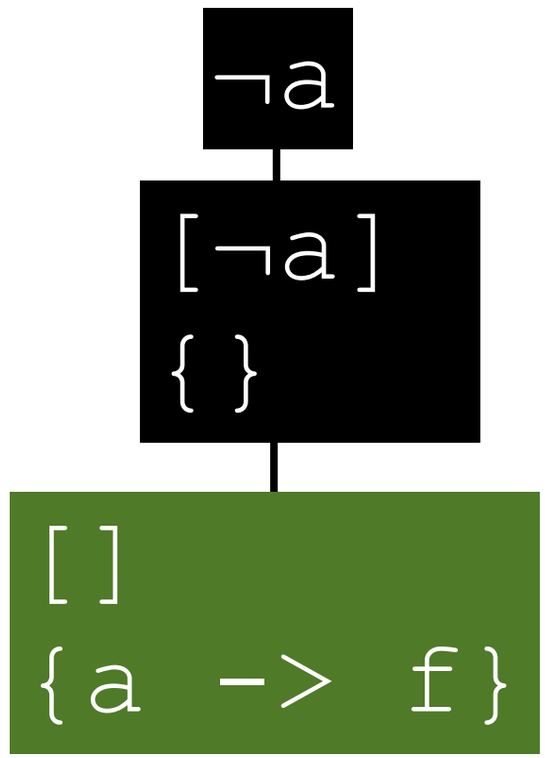
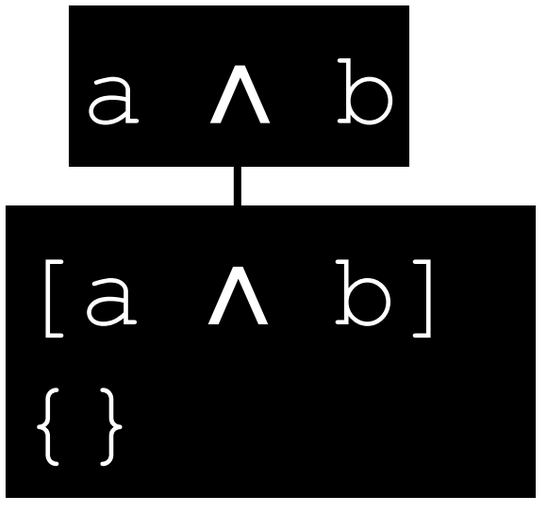


Logical And

$$a \wedge b$$

Logical And



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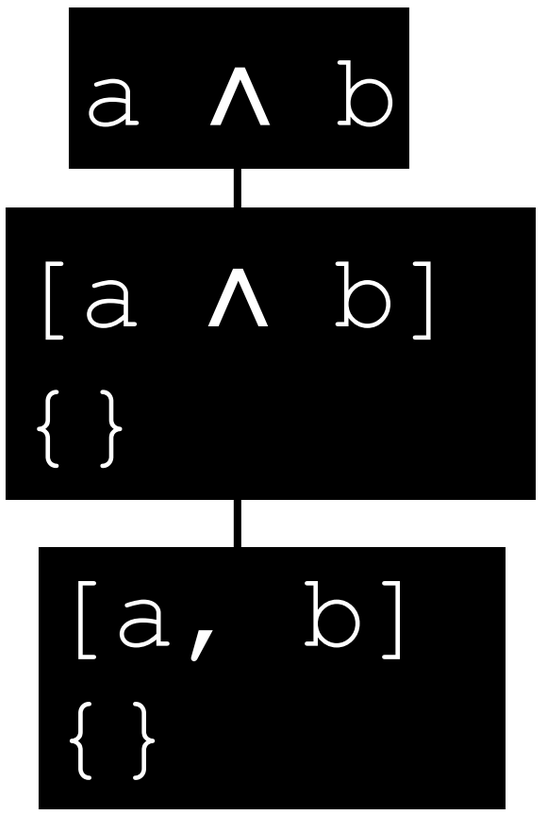

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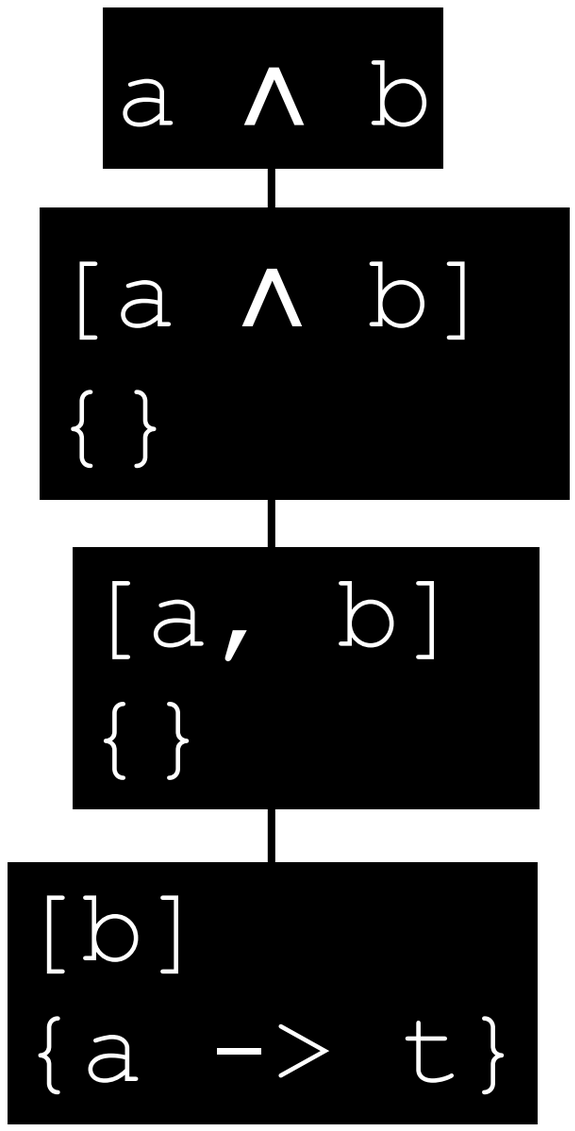

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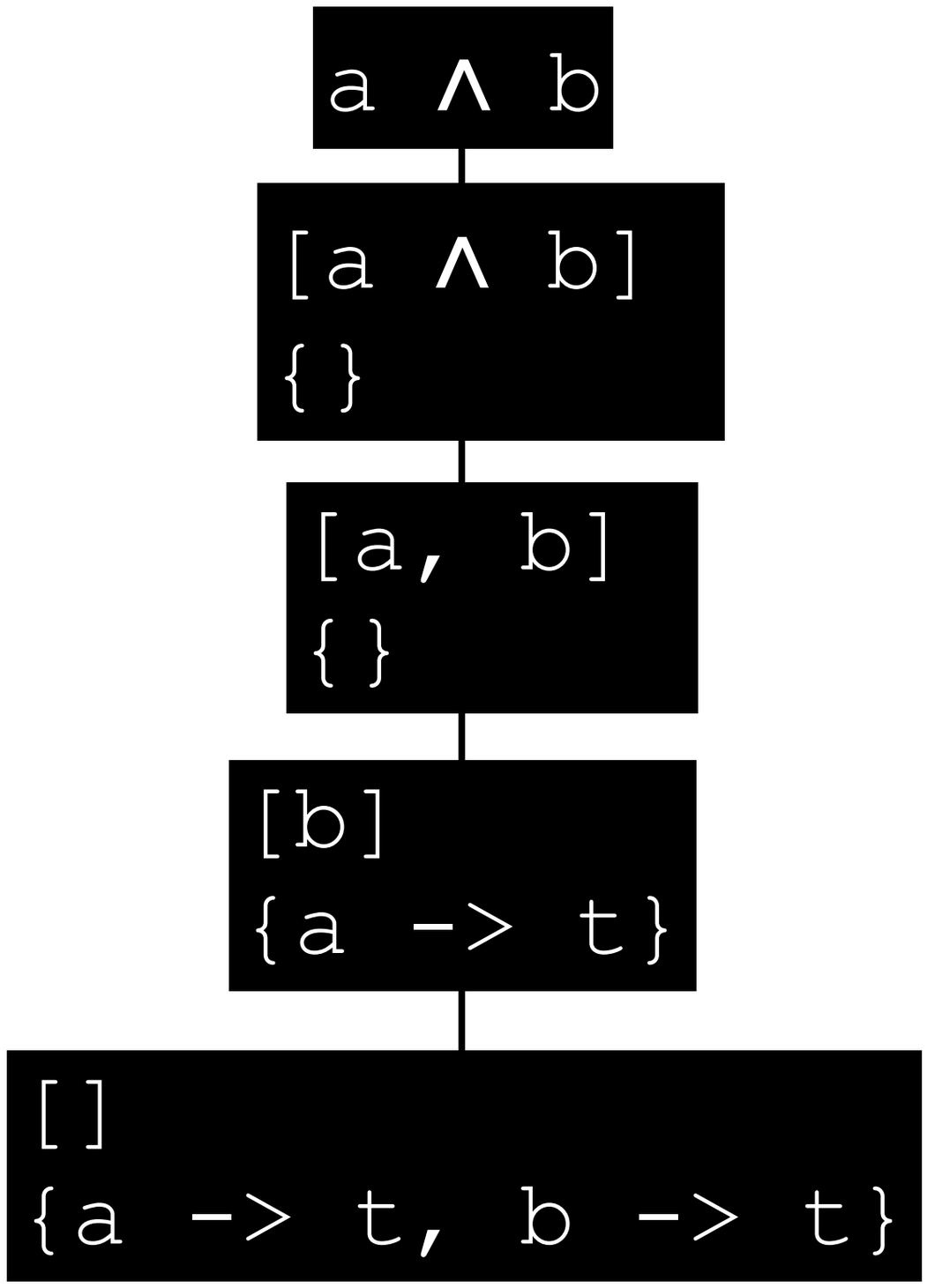

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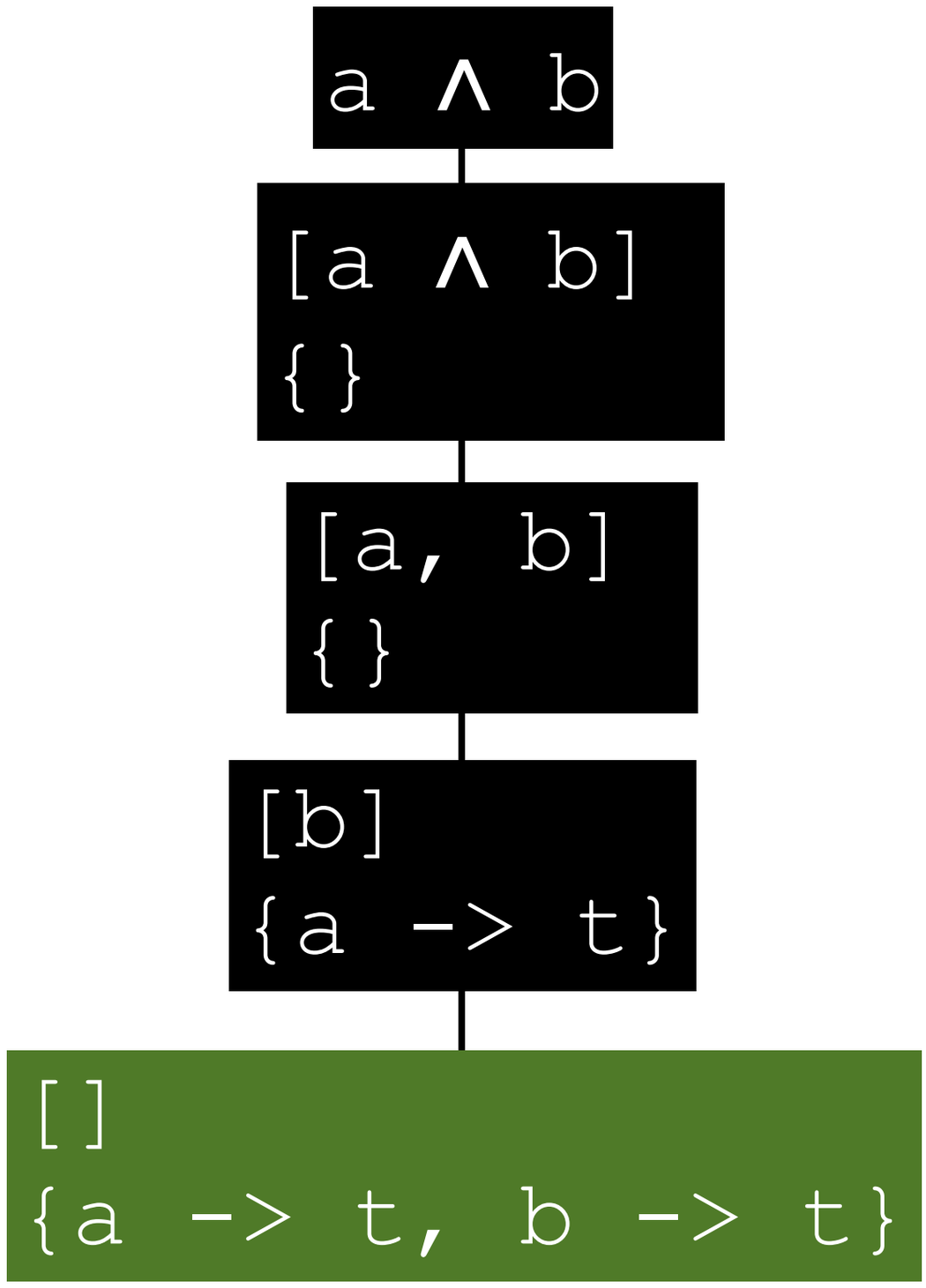
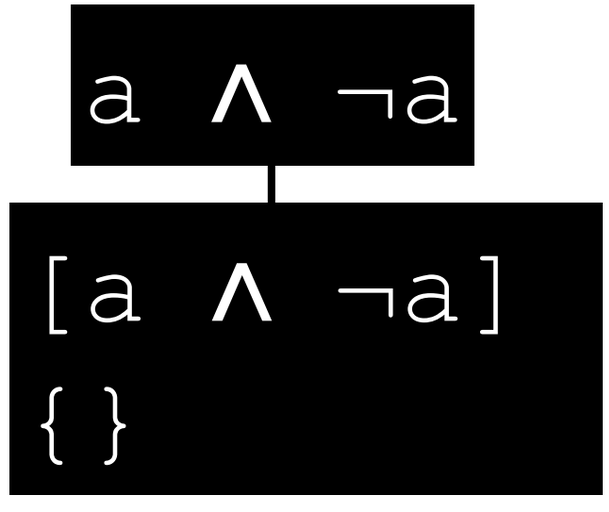


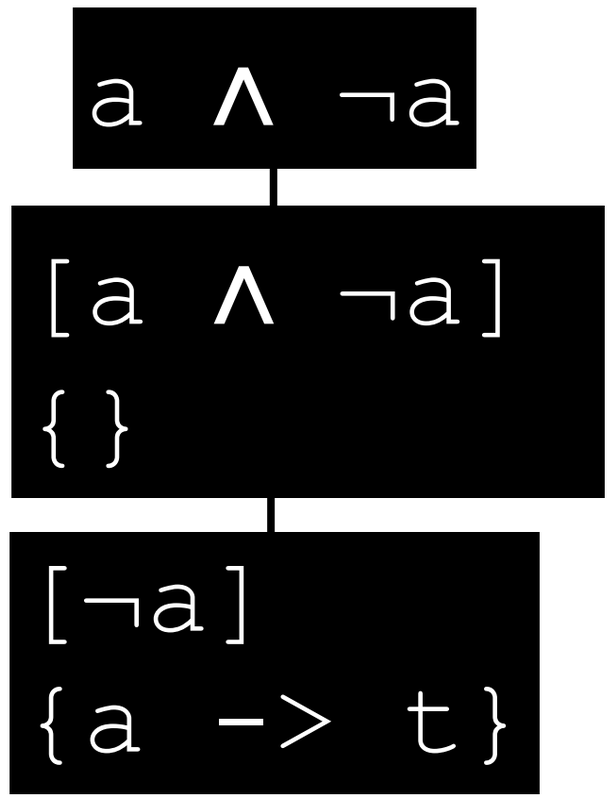
Logical And

$$a \wedge \neg a$$

Logical And



Logical And



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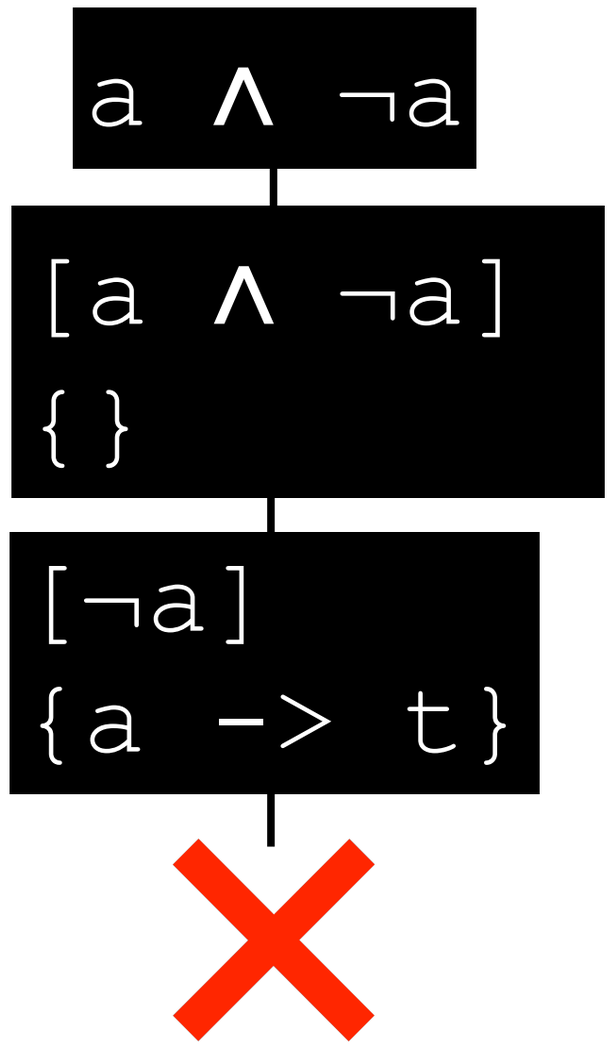
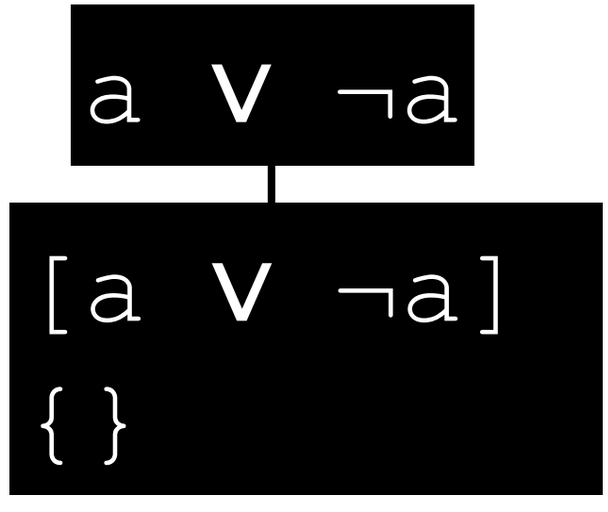


Exercise: First Side of SAT Sheet

Logical Or

$$a \vee \neg a$$

Logical Or



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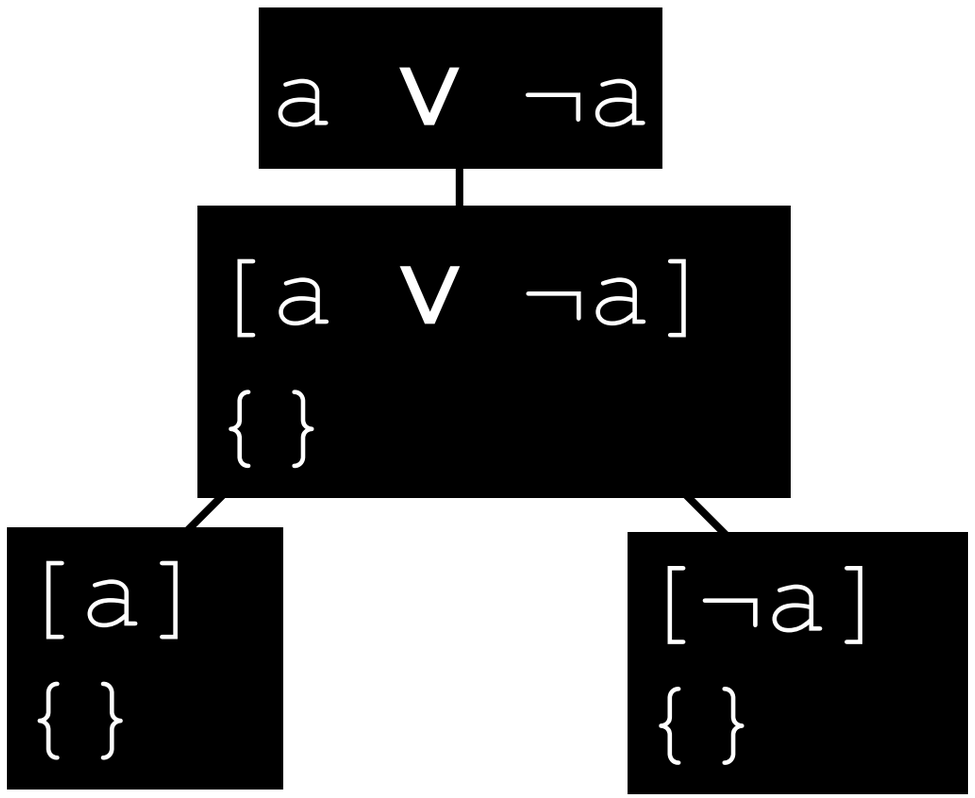

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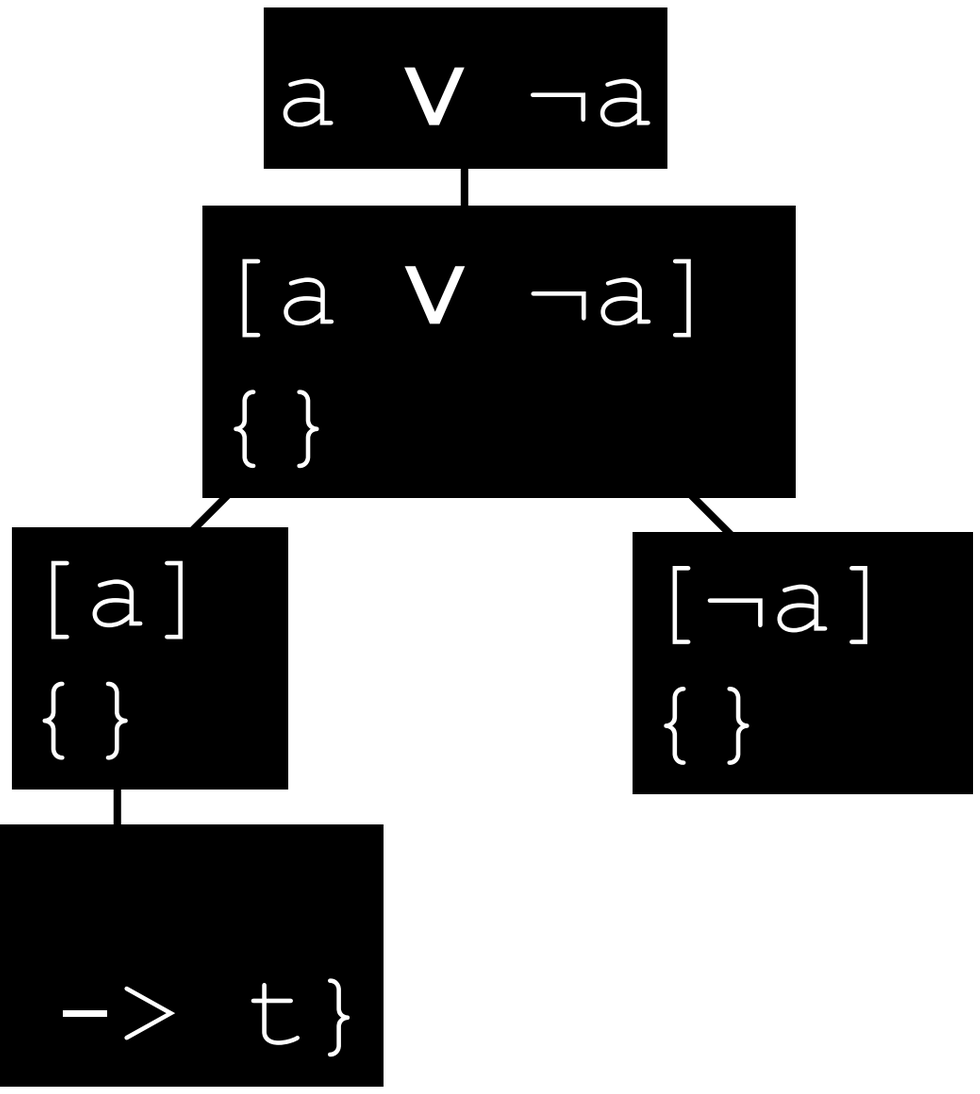

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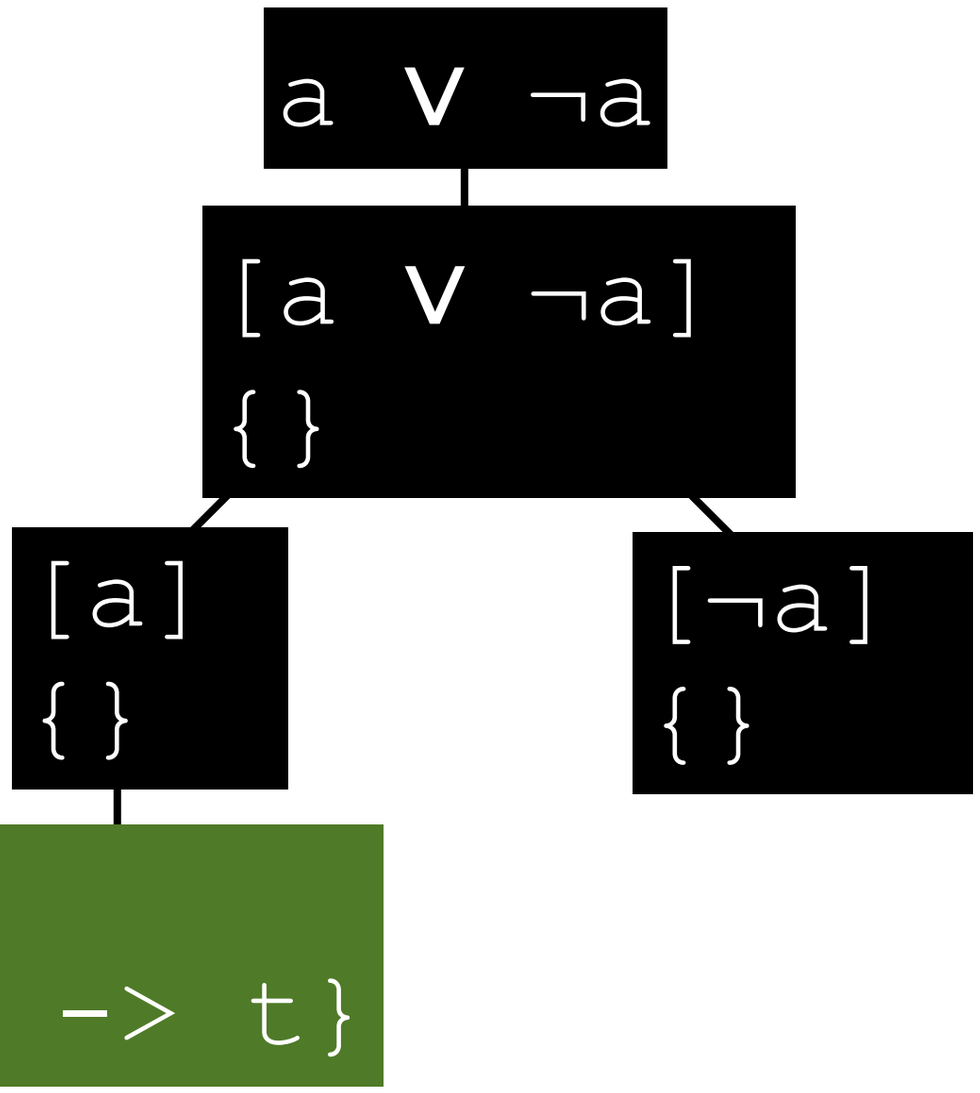
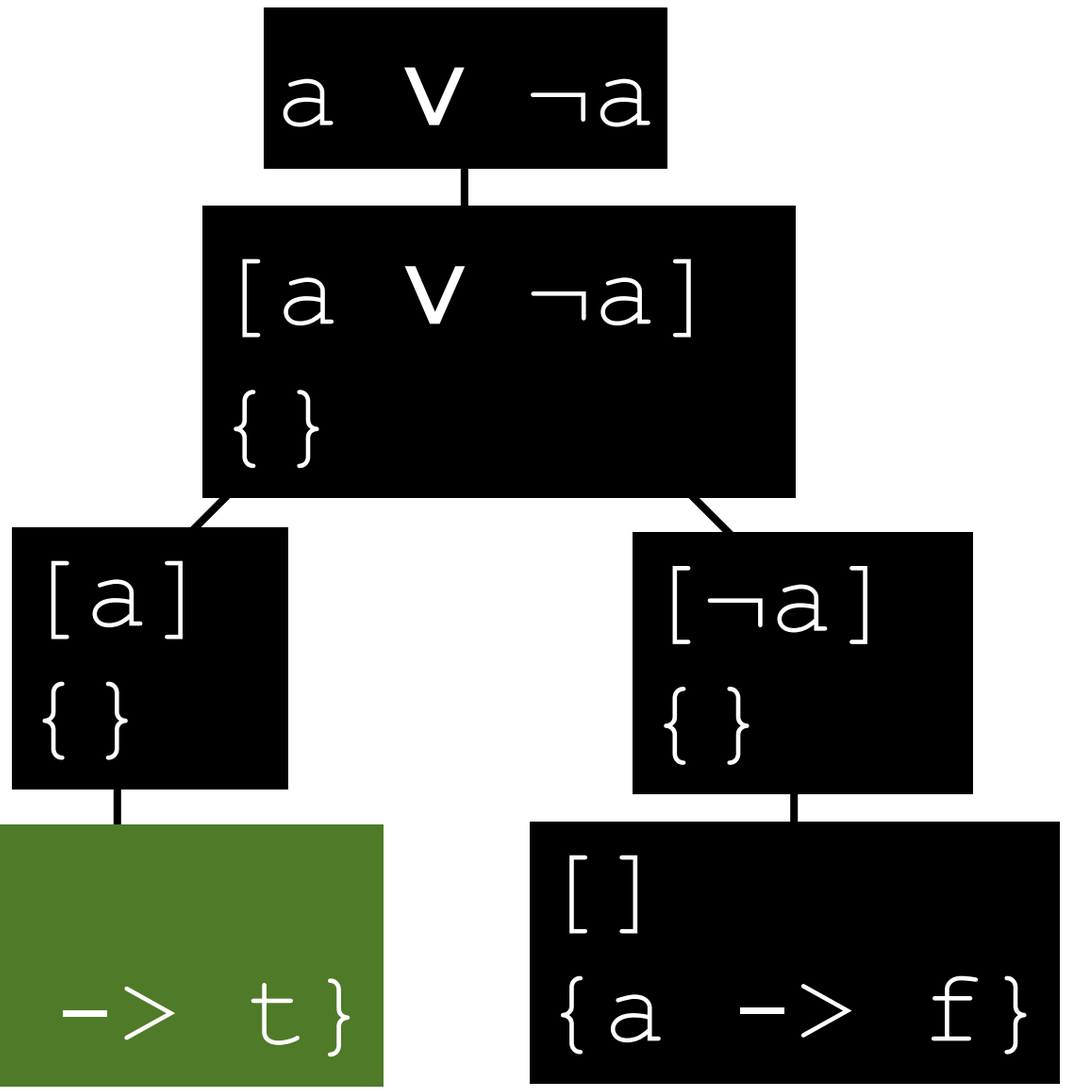
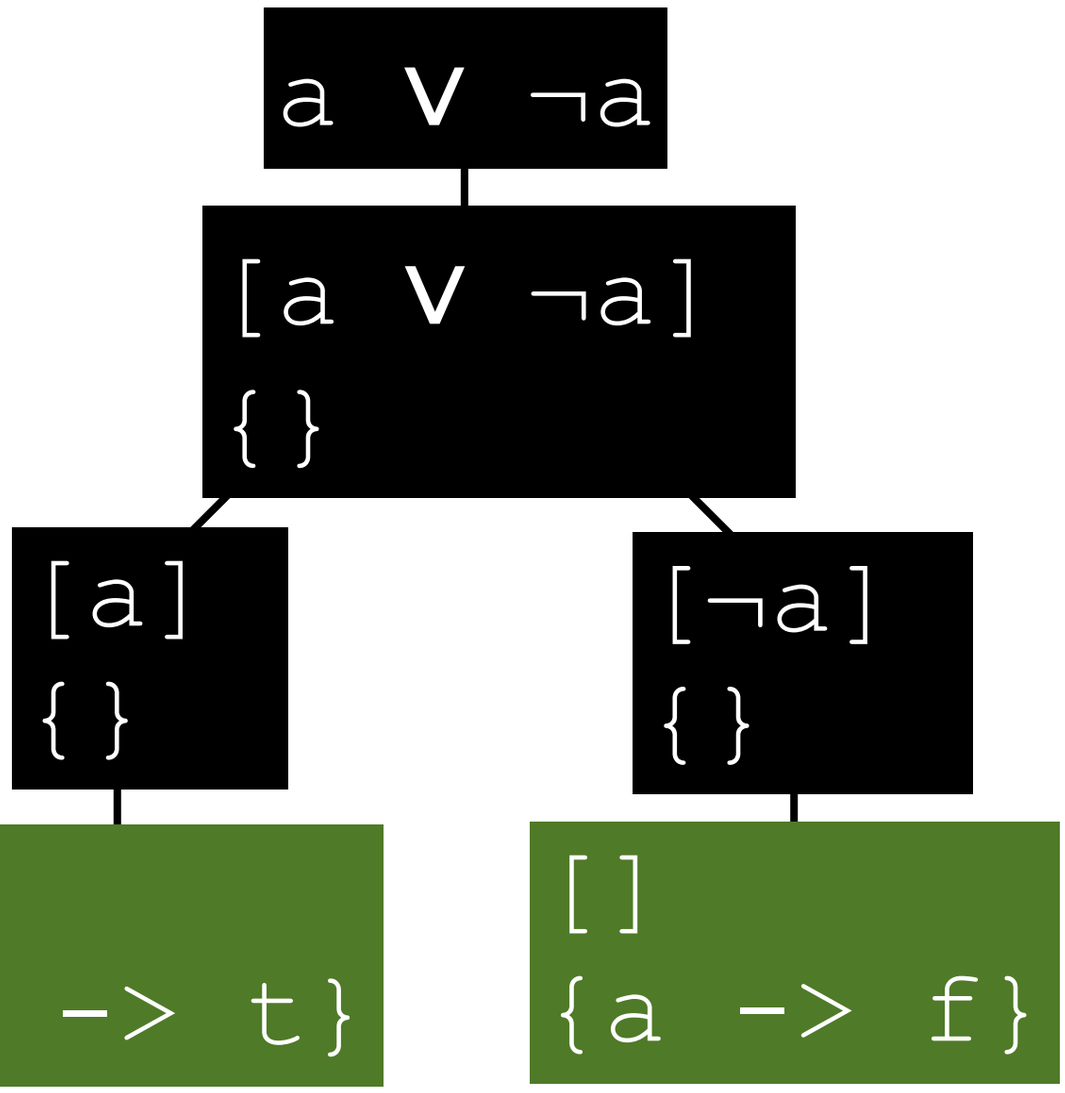


Logical Or



Logical Or

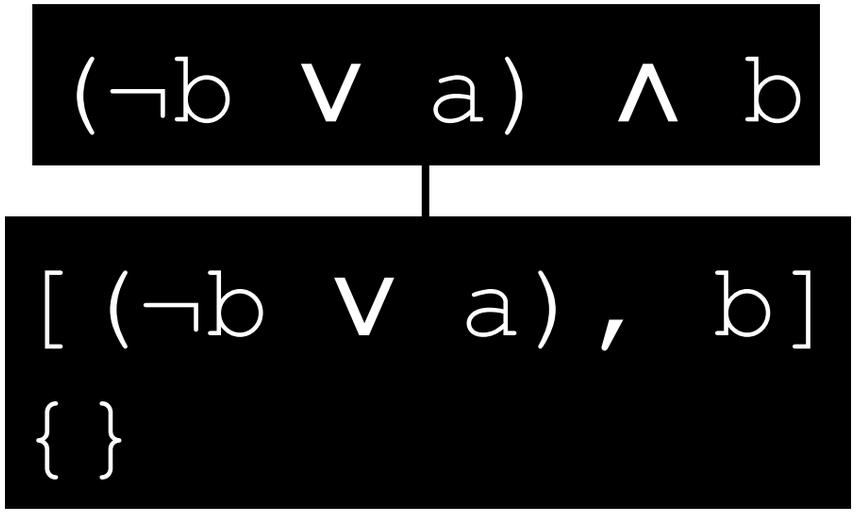


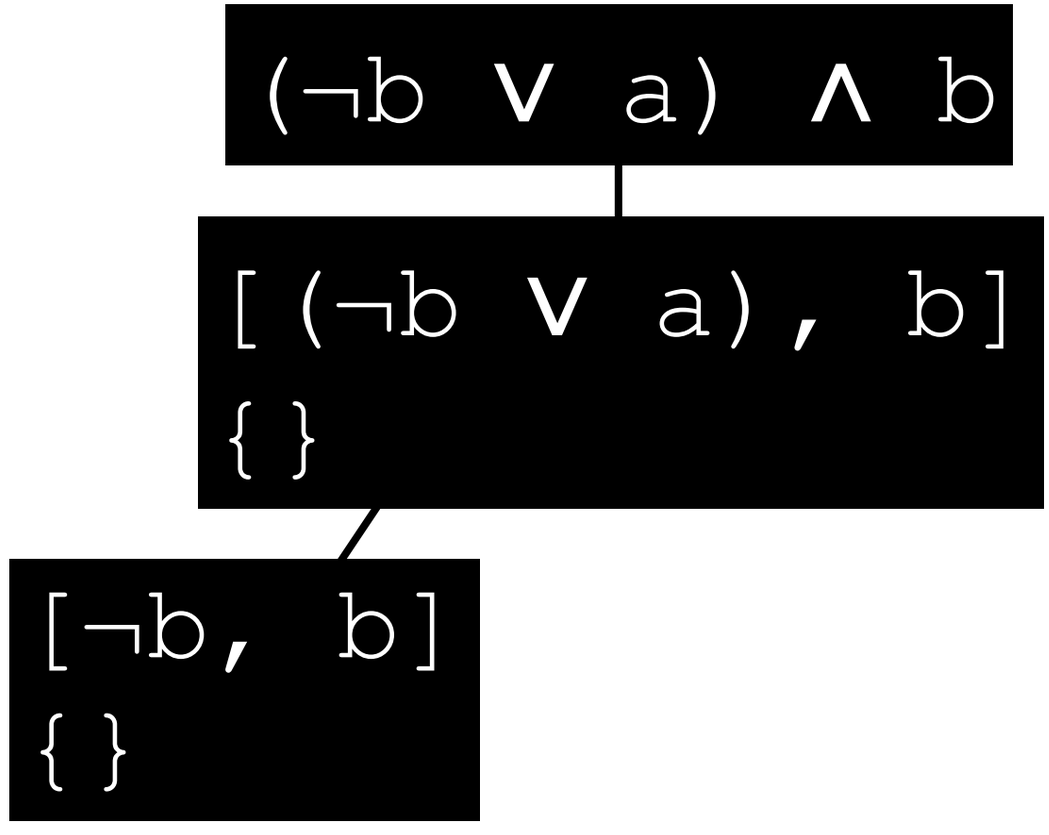
Examples

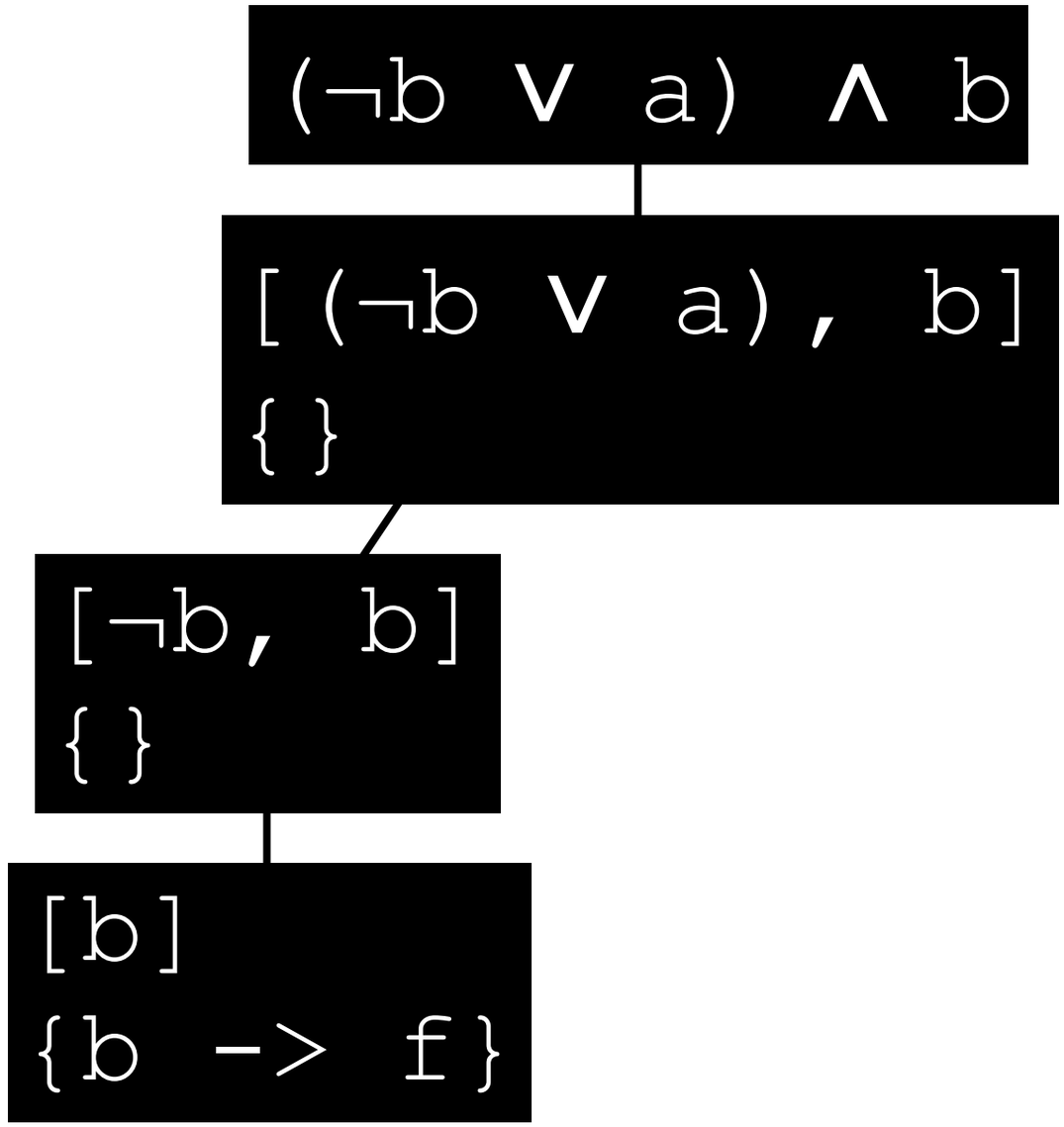
Example 1:

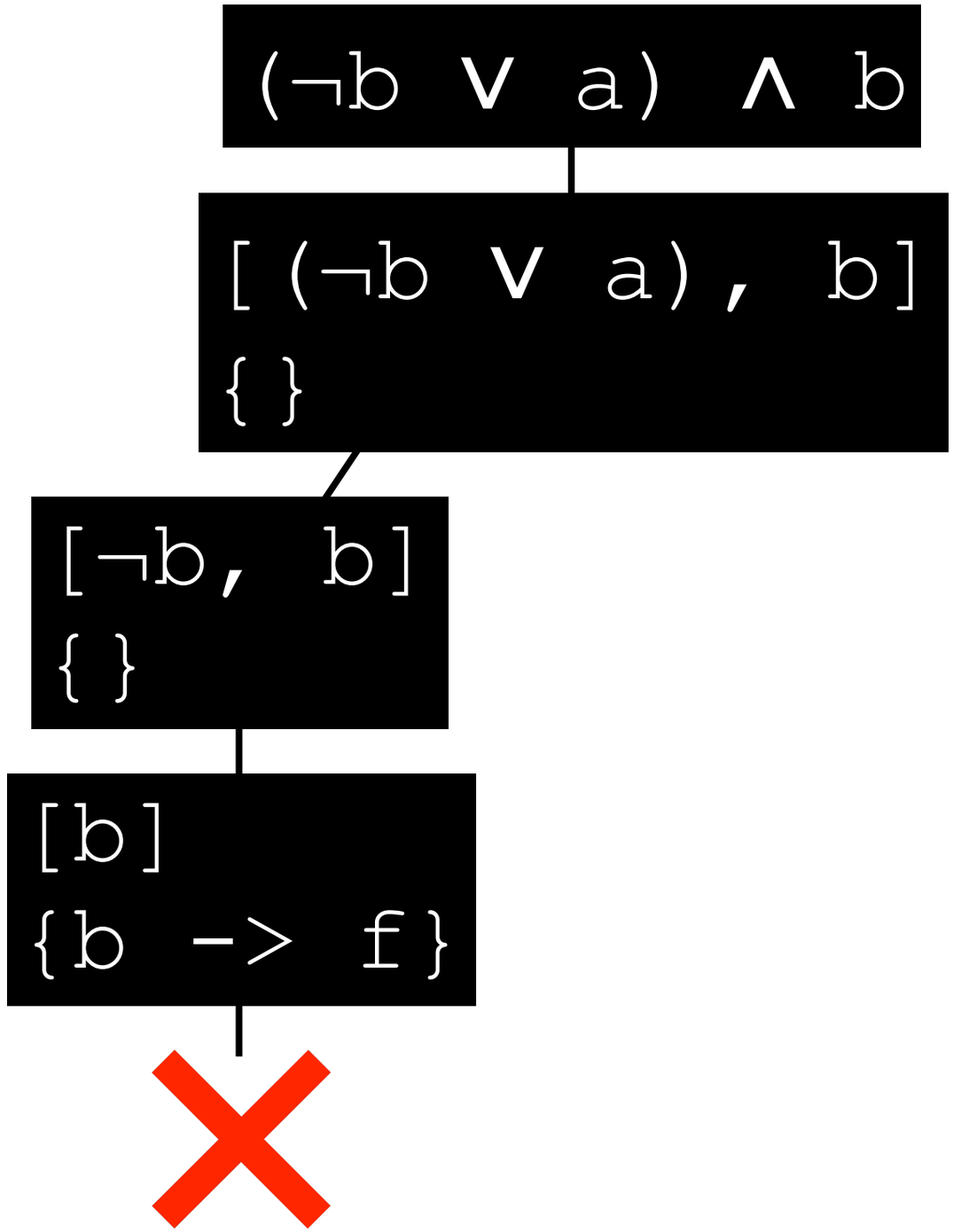
$(\neg b \lor a) \land b$

$$(\neg b \lor a) \land b$$


(¬b ∨ a) ∧ b
[(¬b ∨ a), b]
{}


(¬b ∨ a) ∧ b
[(¬b ∨ a), b]
{}
[¬b, b]
{}


(¬b ∨ a) ∧ b
[(¬b ∨ a), b]
{}
[¬b, b]
{}
[b]
{b -> f}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[¬b, b]
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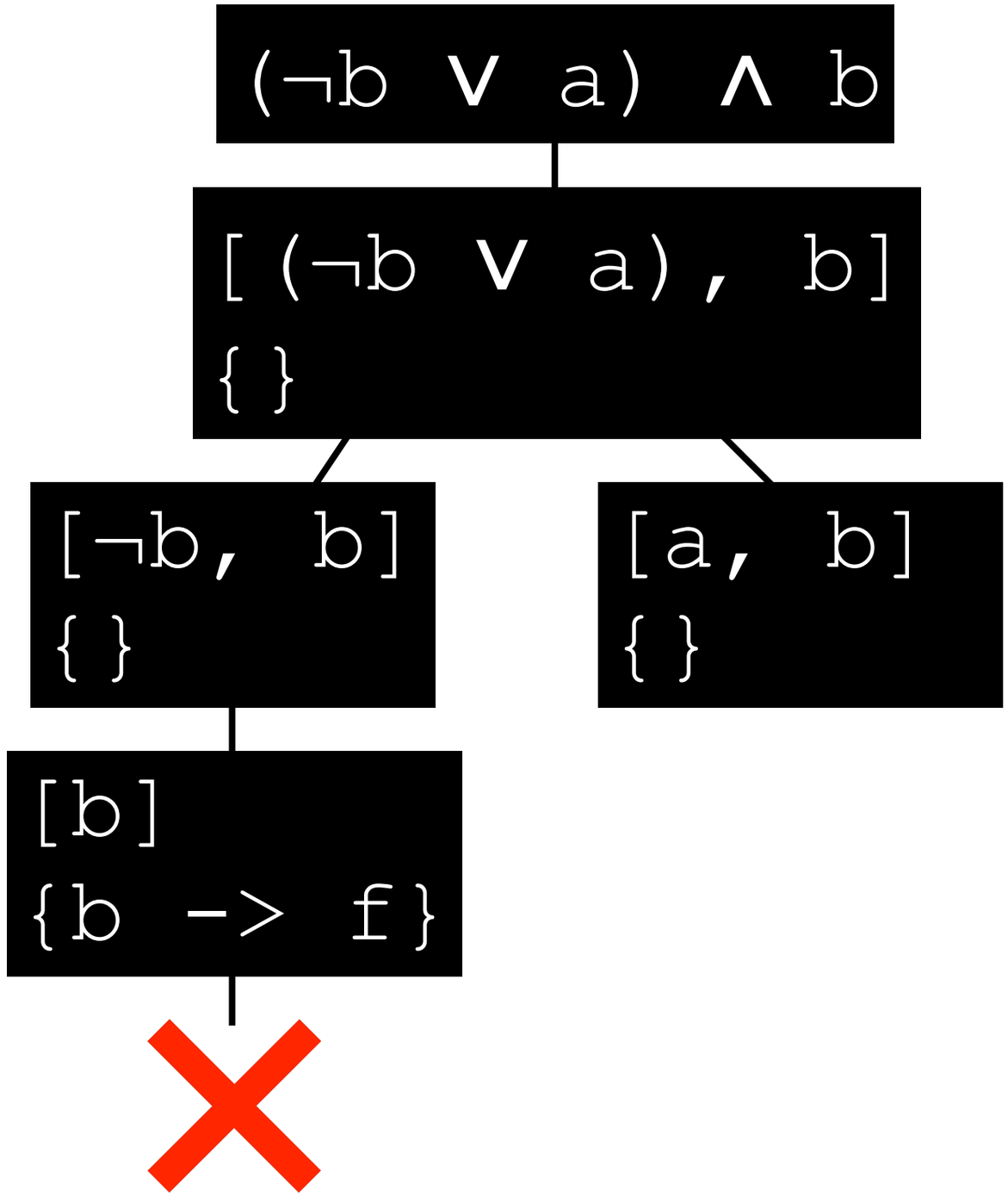
[b]
{b -> f}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[¬b, b]
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[a, b]
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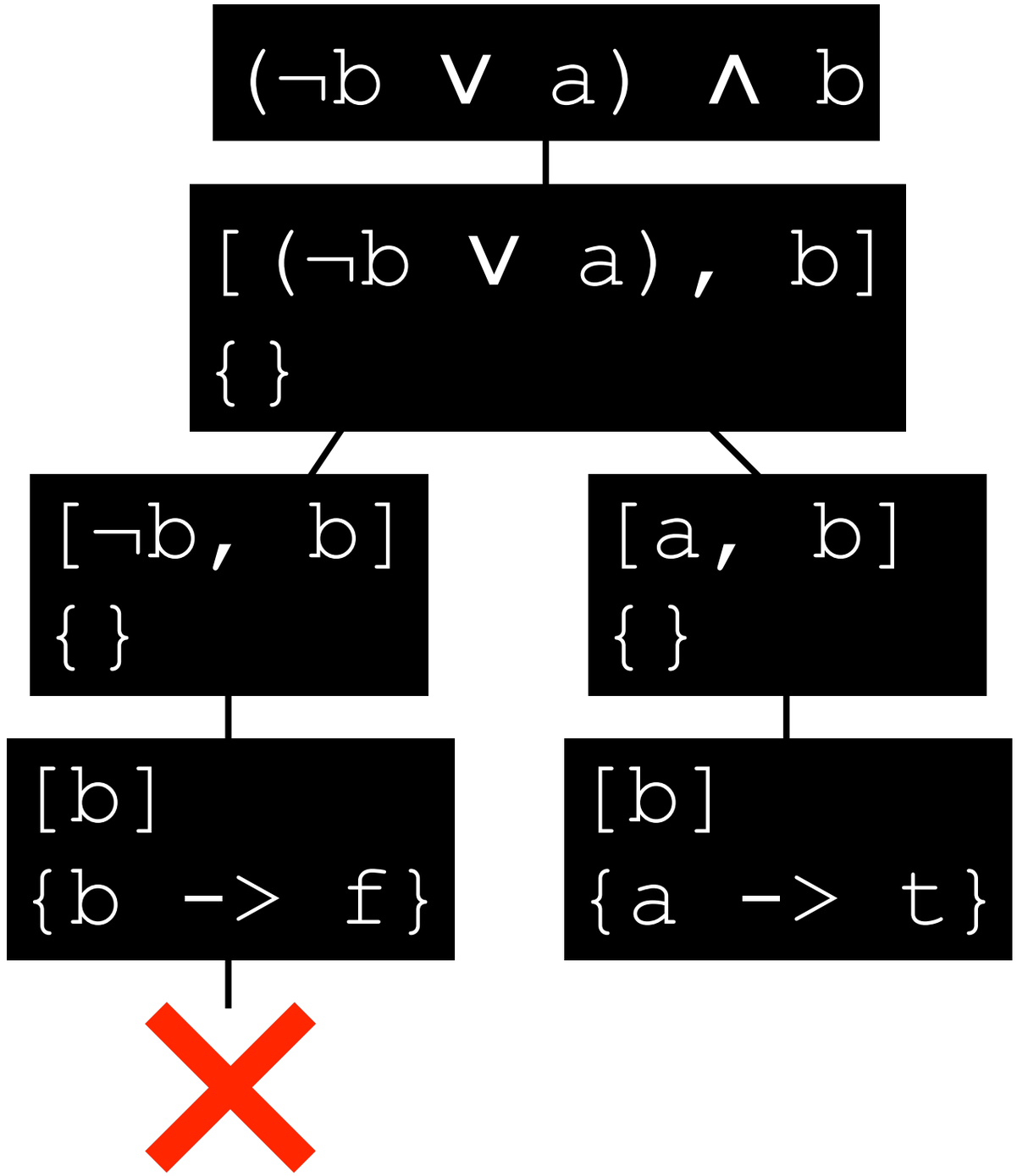
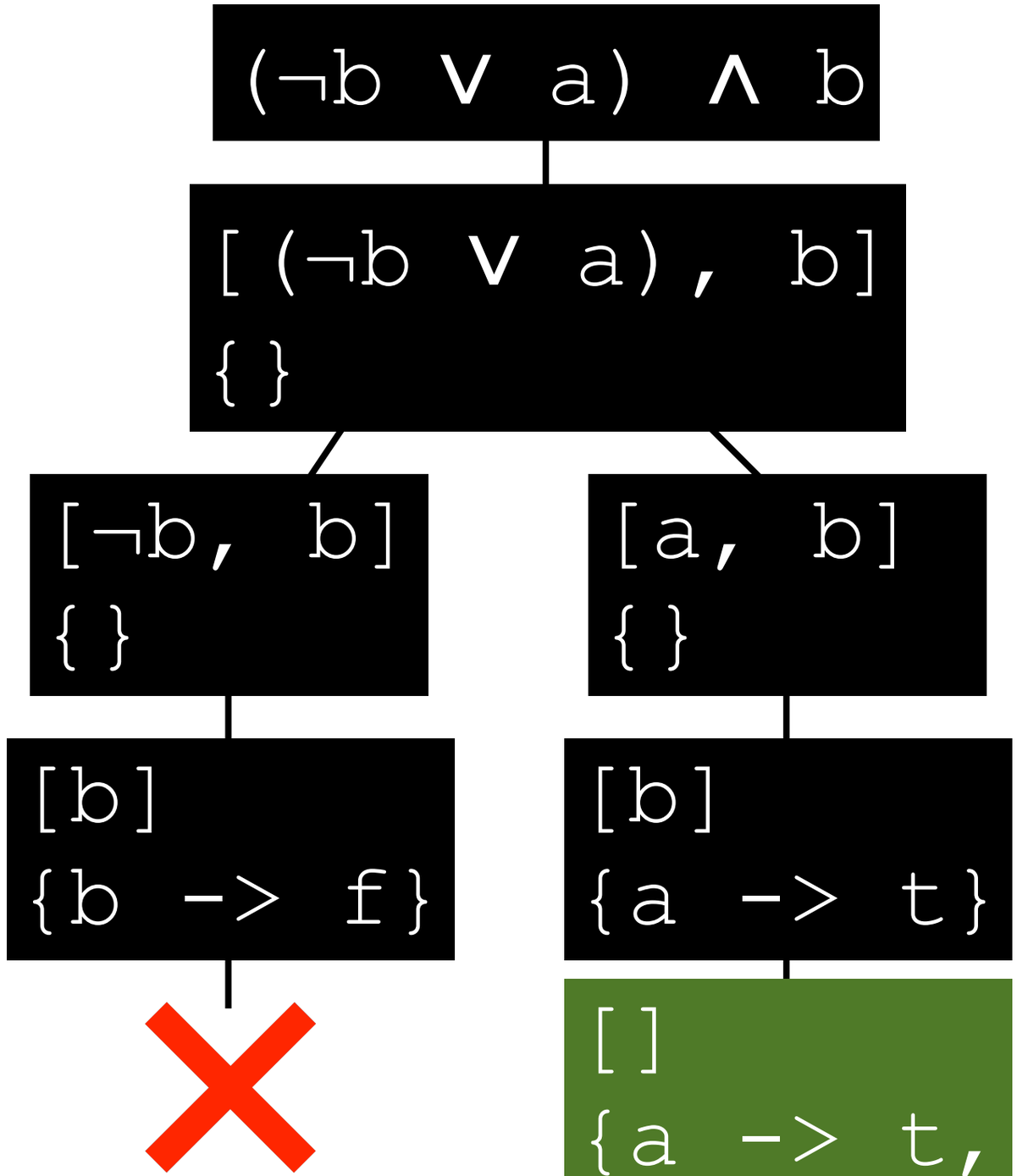
[b]
{b -> f}

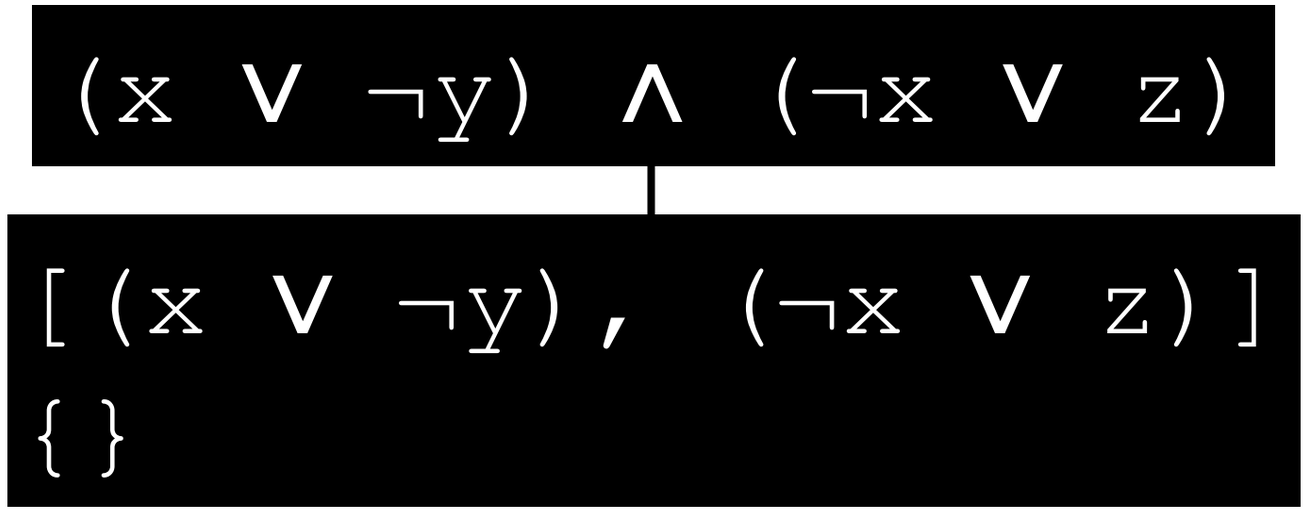

(¬b ∨ a) ∧ b
[(¬b ∨ a), b]
{}
[¬b, b]
{}
[a, b]
{}
[b]
{b -> f}
[b]
{a -> t}

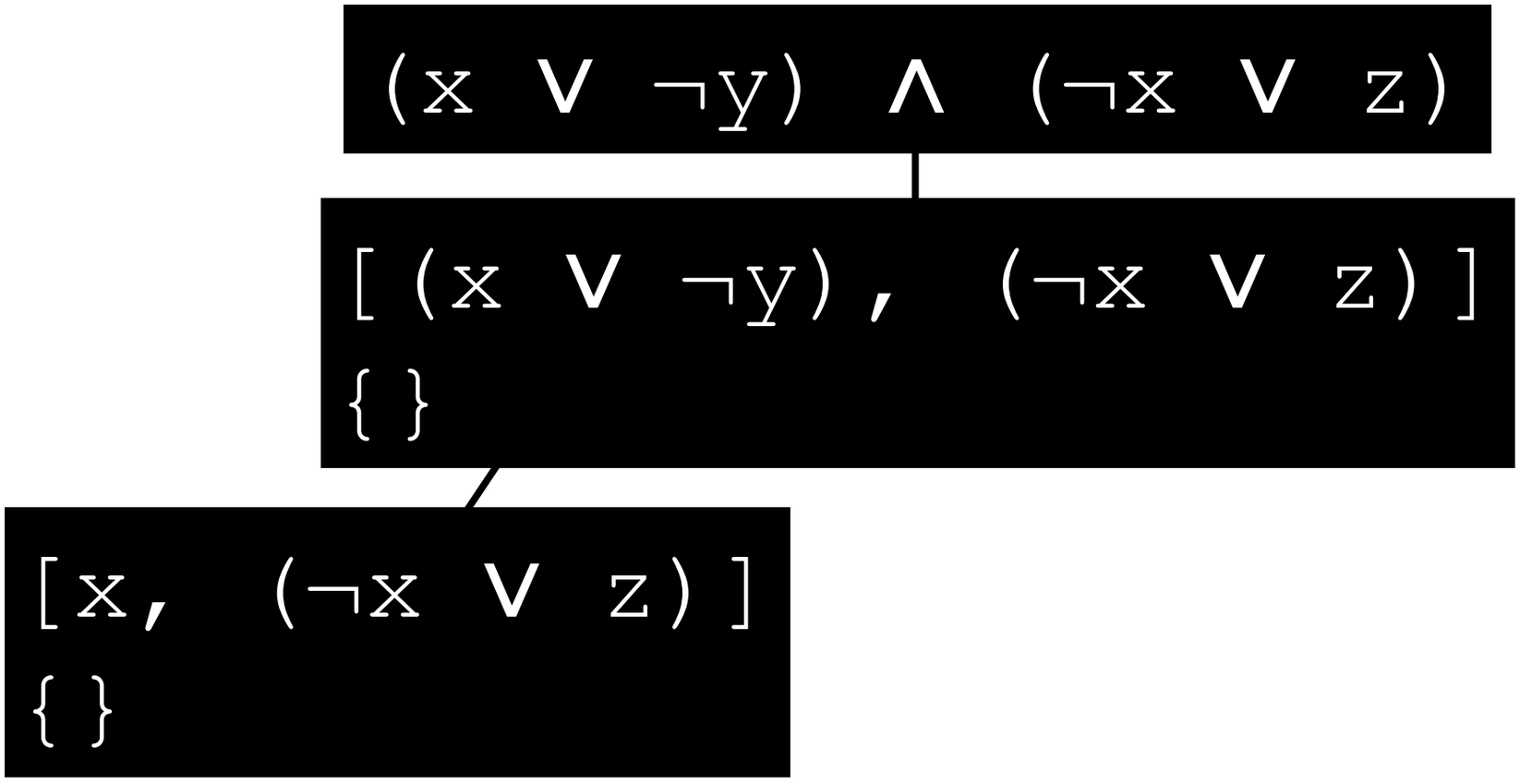

(¬b ∨ a) ∧ b
[(¬b ∨ a), b]
{}
[¬b, b]
{}
[a, b]
{}
[b]
{b -> f}
[b]
{a -> t}
[]
{a -> t,
b -> t}

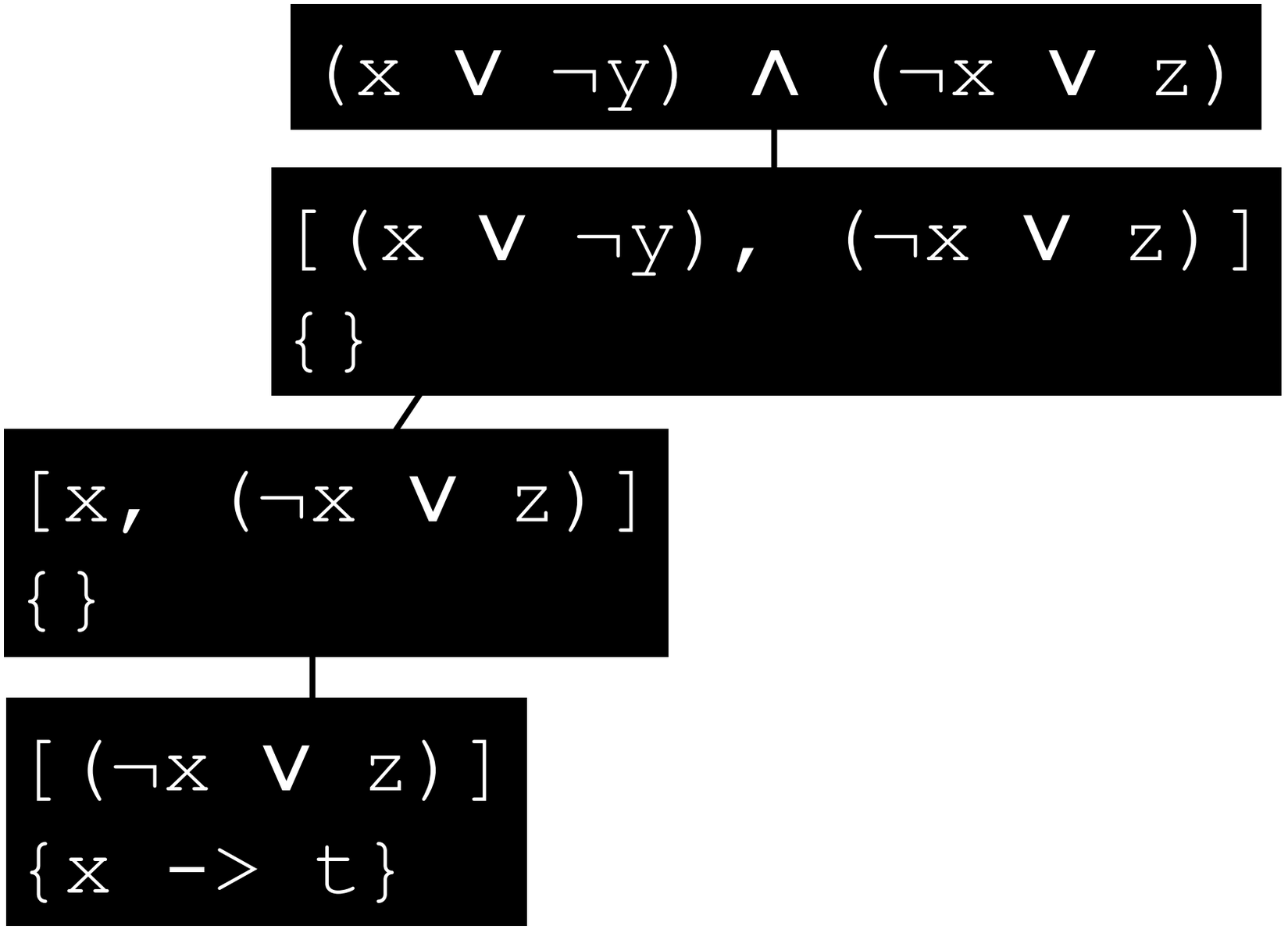
Example 2:

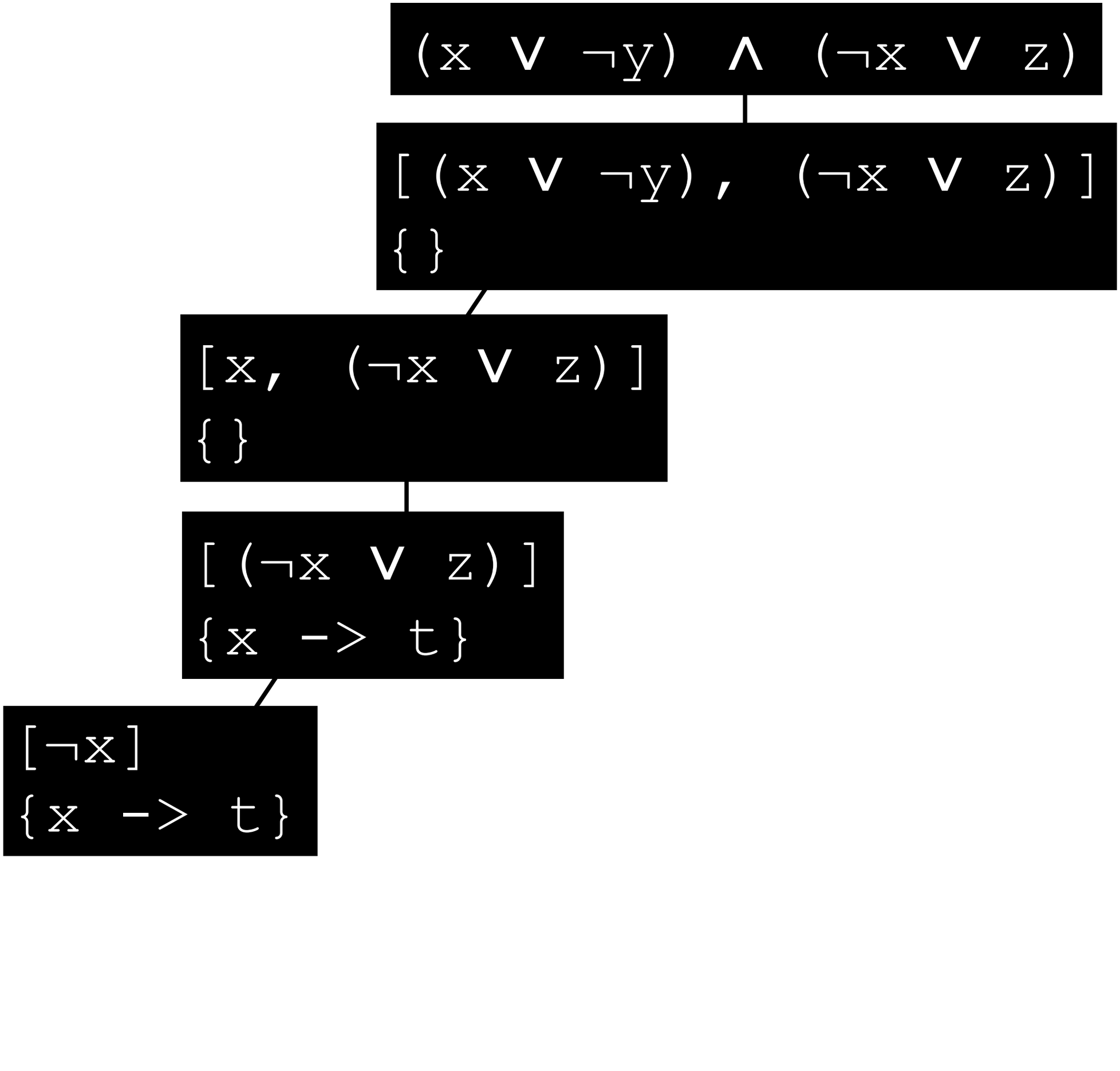
$$(x \lor \neg y) \land (\neg x \lor z)$$

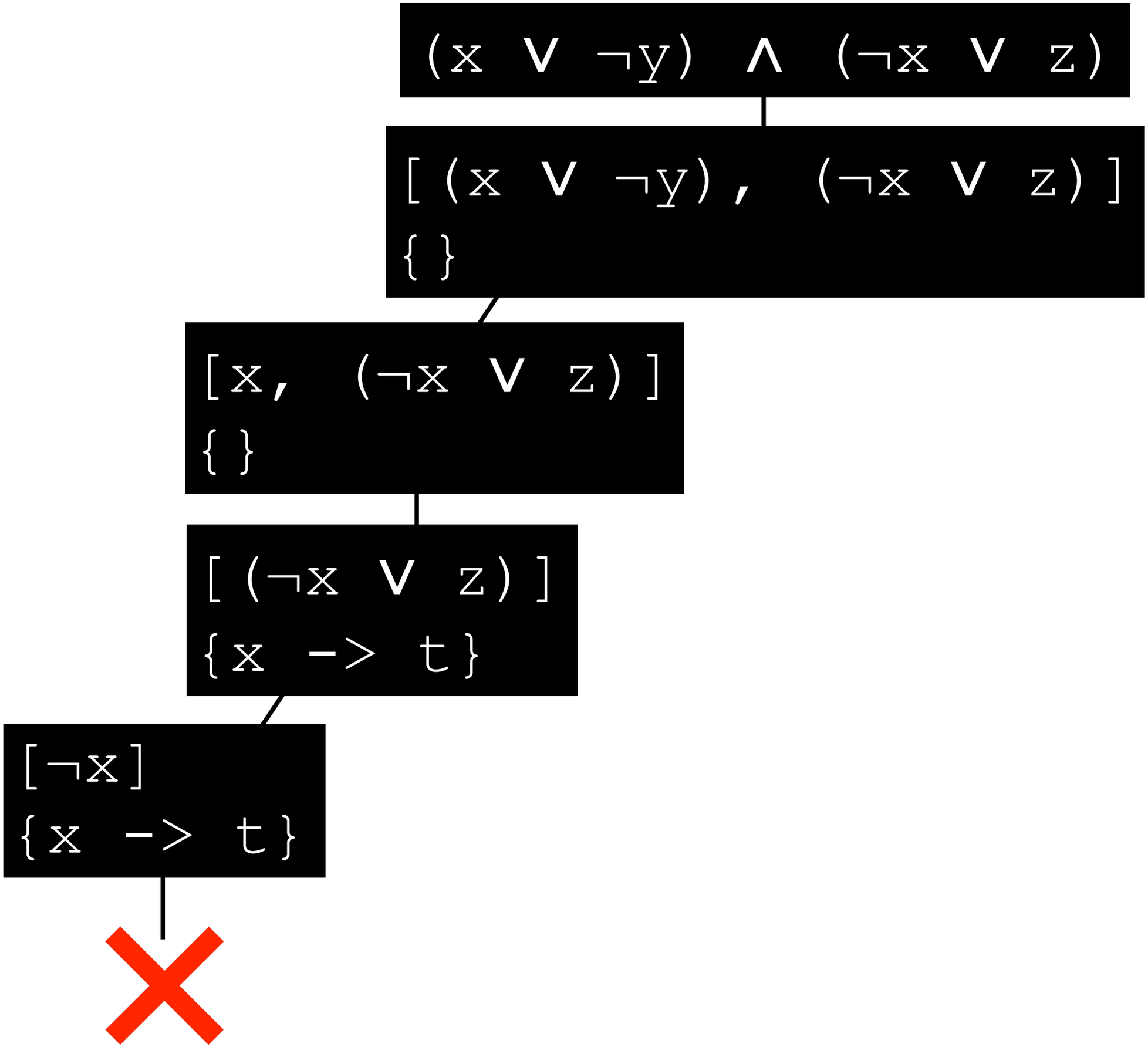
$$(x \lor \neg y) \land (\neg x \lor z)$$

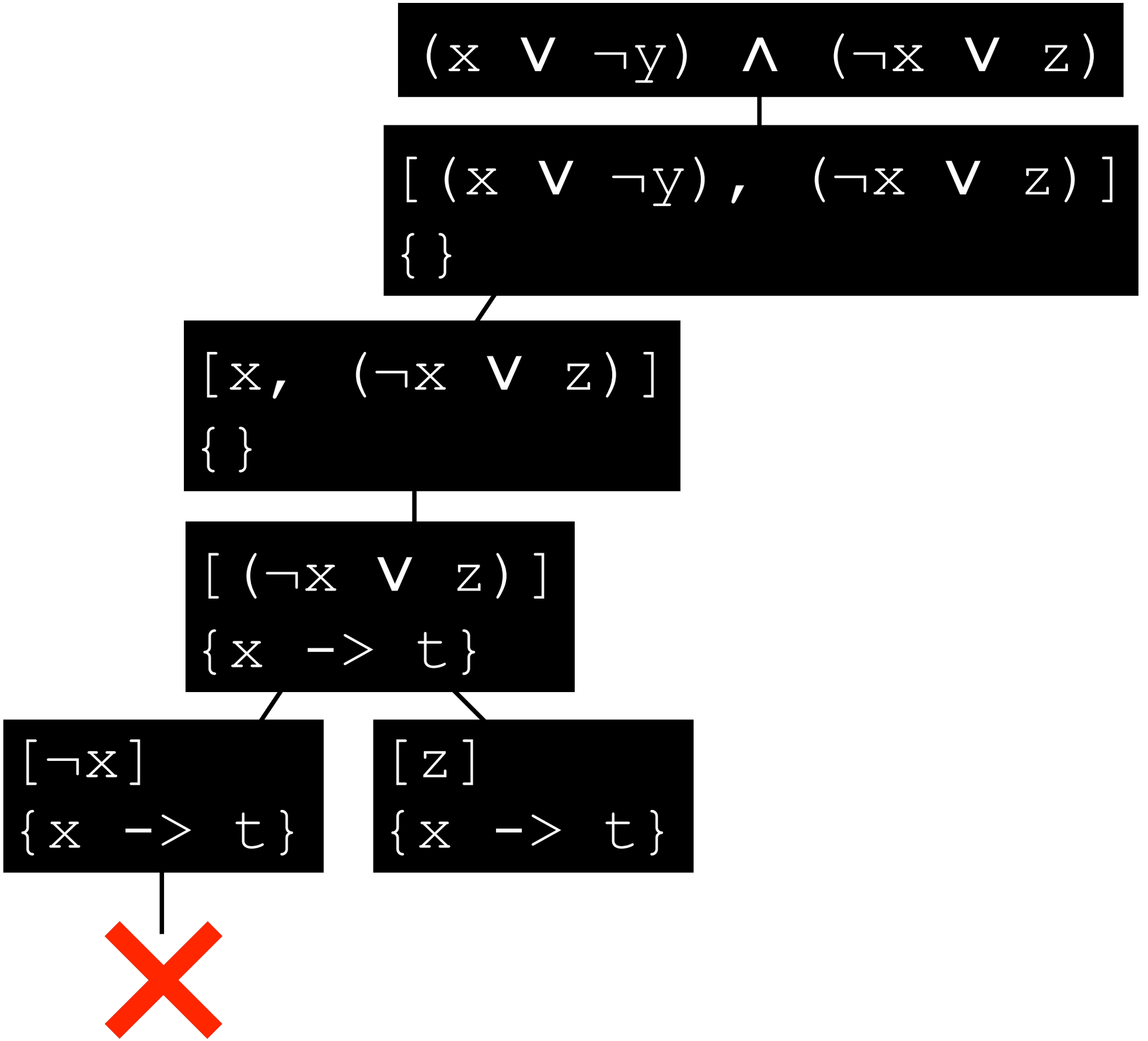

(x ∨ ¬y) ∧ (¬x ∨ z)
[(x ∨ ¬y), (¬x ∨ z)]
{}

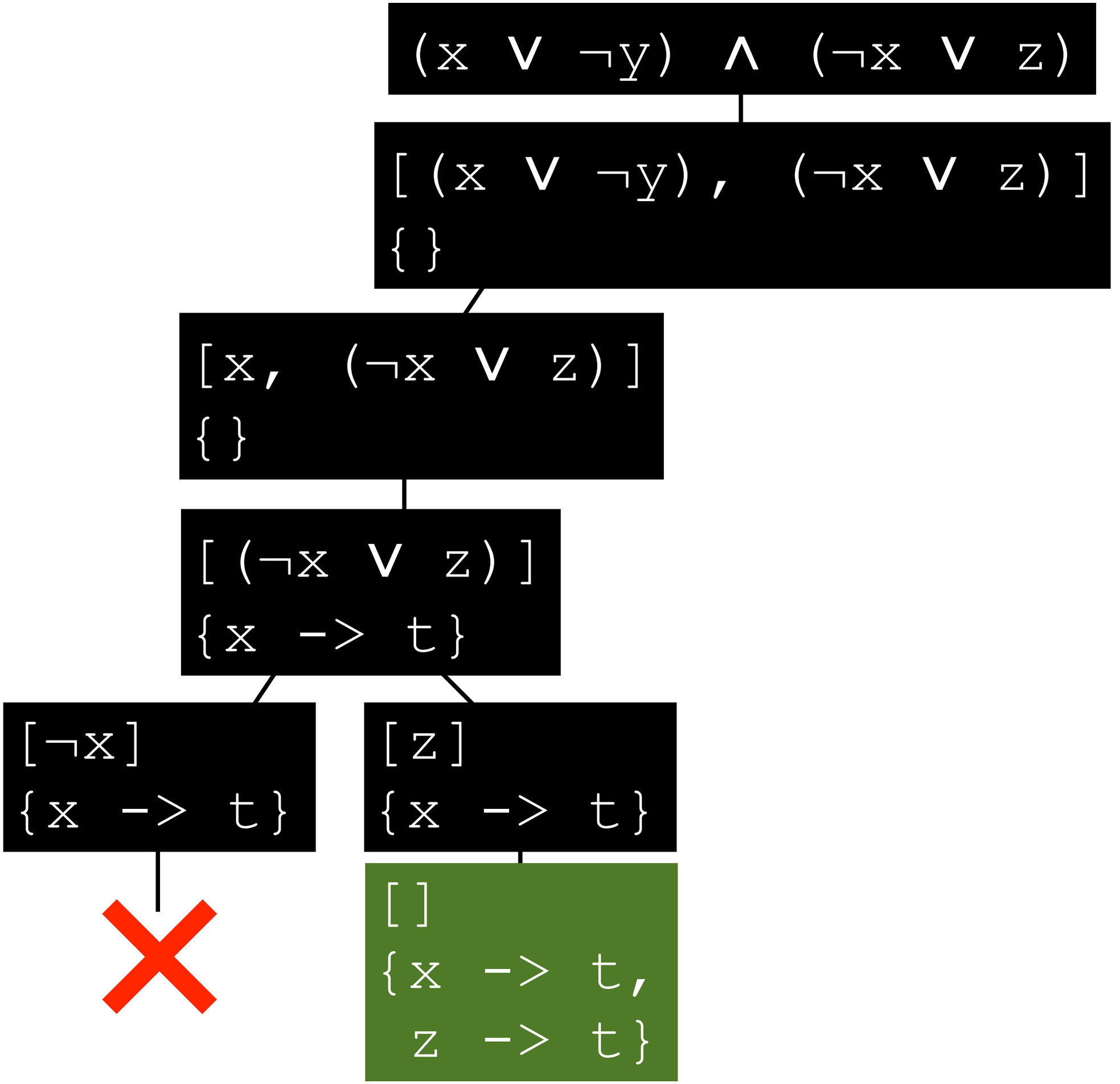

(x ∨ ¬y) ∧ (¬x ∨ z)
[(x ∨ ¬y), (¬x ∨ z)]
{}
[x, (¬x ∨ z)]
{}

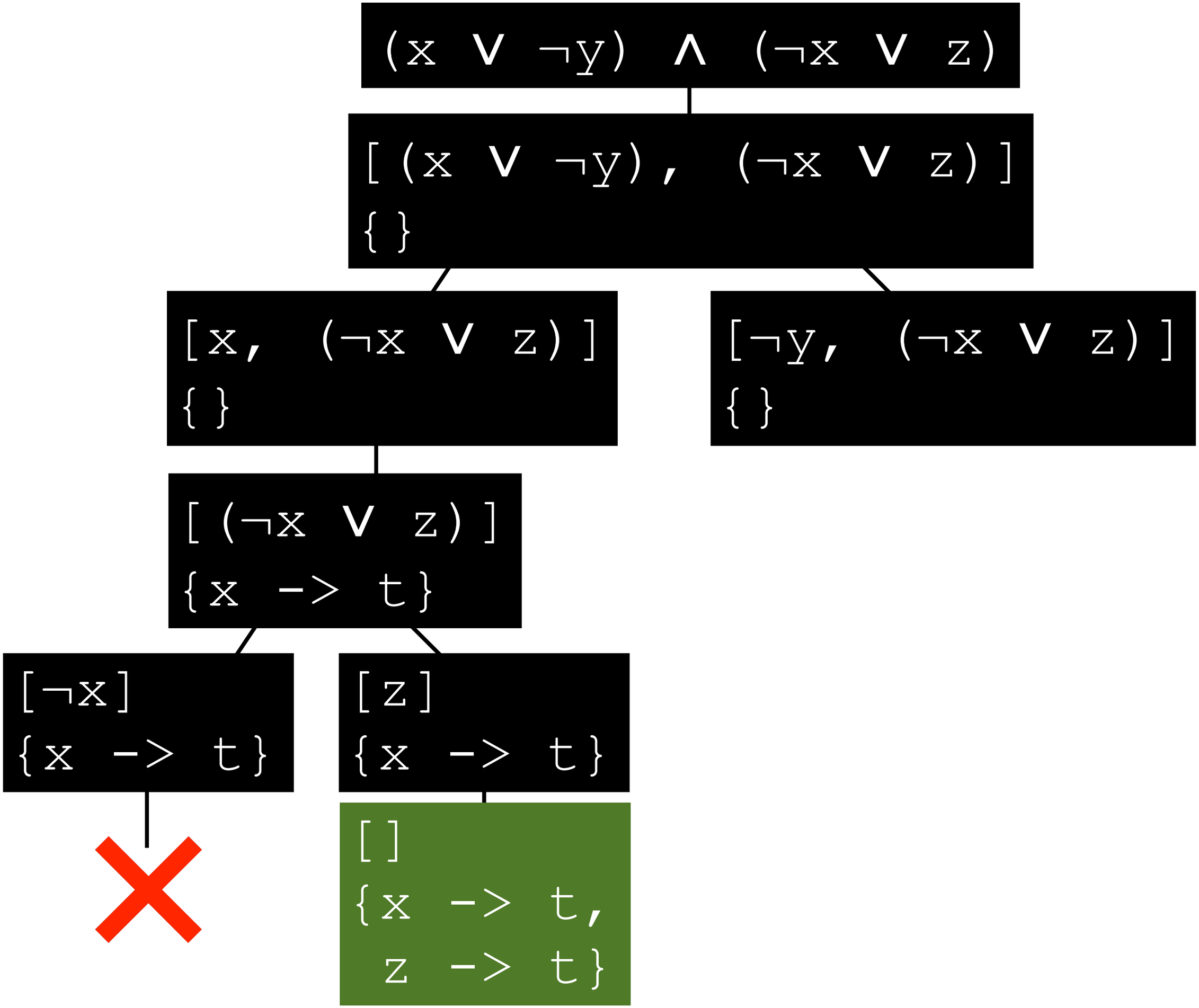

(x ∨ ¬y) ∧ (¬x ∨ z)
[(x ∨ ¬y), (¬x ∨ z)]
{}
[x, (¬x ∨ z)]
{}
[(¬x ∨ z)]
{x -> t}


(x ∨ ¬y) ∧ (¬x ∨ z)
[(x ∨ ¬y), (¬x ∨ z)]
{}
[x, (¬x ∨ z)]
{}
[(¬x ∨ z)]
{x -> t}
[¬x]
{x -> t}


(x ∨ ¬y) ∧ (¬x ∨ z)
[(x ∨ ¬y), (¬x ∨ z)]
{}
[x, (¬x ∨ z)]
{}
[(¬x ∨ z)]
{x -> t}
[¬x]
{x -> t}


(x ∨ ¬y) ∧ (¬x ∨ z)
[(x ∨ ¬y), (¬x ∨ z)]
{}
[x, (¬x ∨ z)]
{}
[(¬x ∨ z)]
{x -> t}
[¬x]
{x -> t}
[z]
{x -> t}


(x ∨ ¬y) ∧ (¬x ∨ z)
[(x ∨ ¬y), (¬x ∨ z)]
{}
[x, (¬x ∨ z)]
{}
[(¬x ∨ z)]
{x -> t}
[¬x]
{x -> t}
[z]
{x -> t}
[]
{x -> t,
z -> t}


(x ∨ ¬y) ∧ (¬x ∨ z)
[(x ∨ ¬y), (¬x ∨ z)]
{}
[x, (¬x ∨ z)]
{}
[¬y, (¬x ∨ z)]
{}
[(¬x ∨ z)]
{x -> t}
[¬x]
{x -> t}
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{x -> t}
[]
{x -> t,
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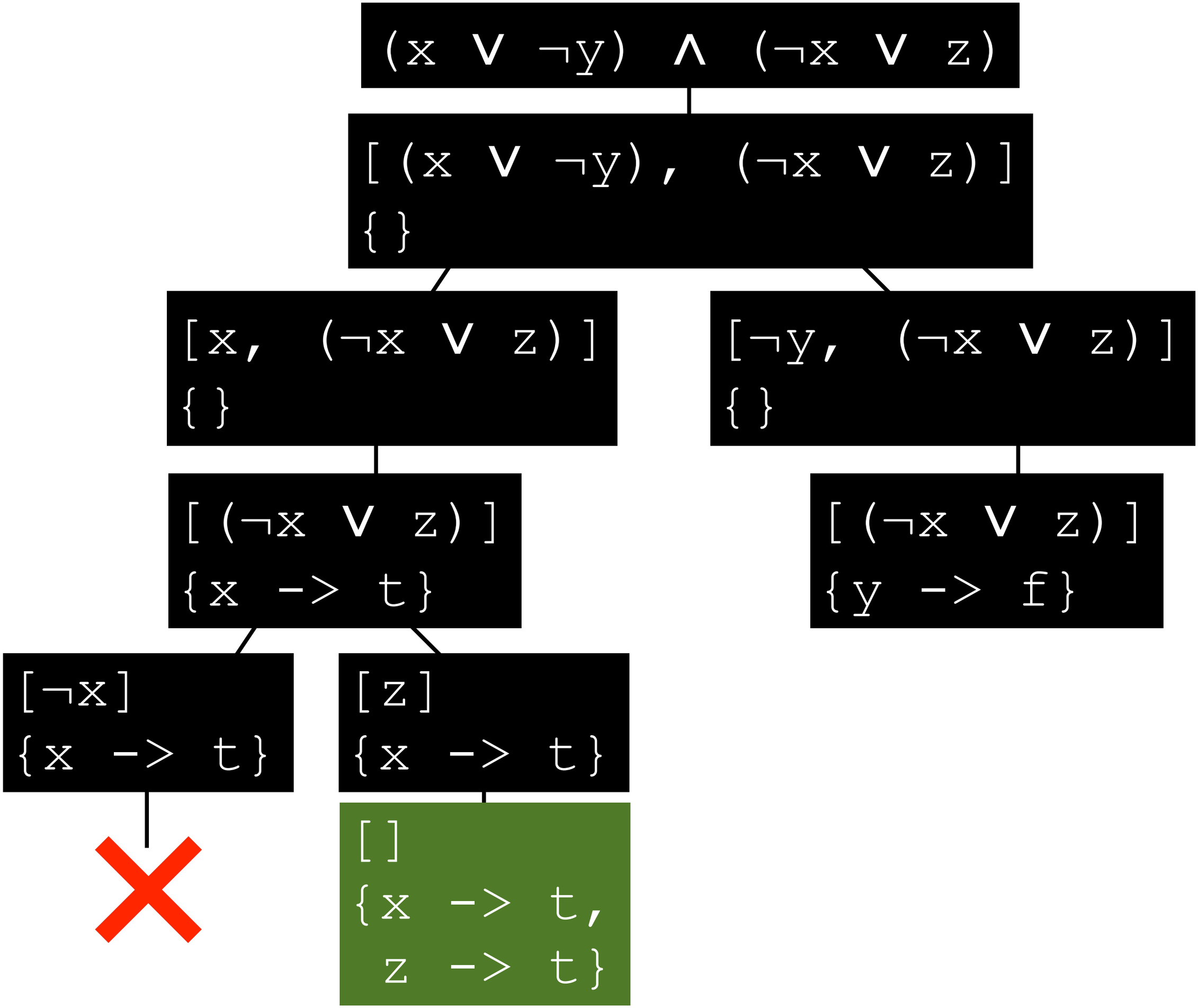
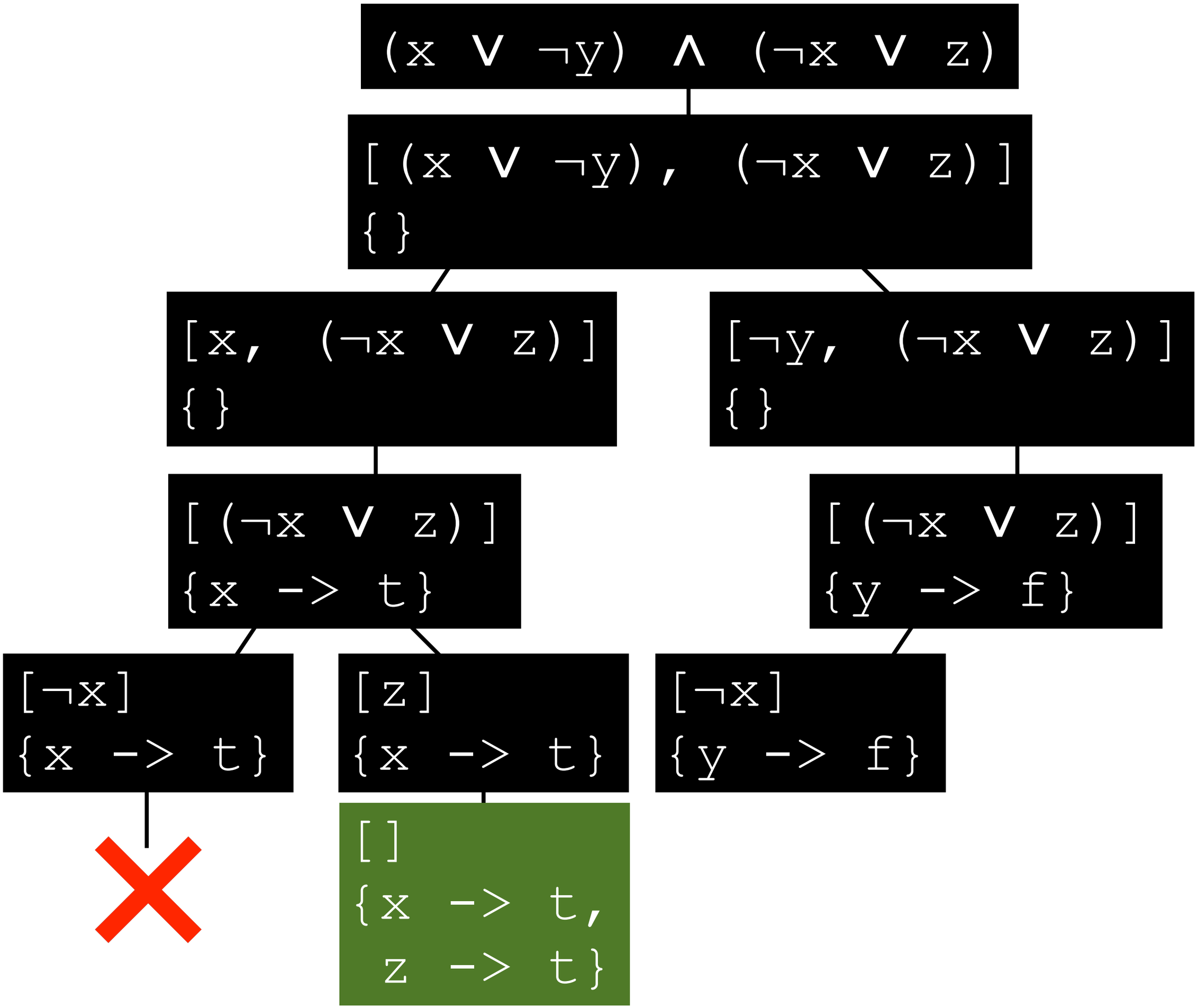

(x ∨ ¬y) ∧ (¬x ∨ z)
[(x ∨ ¬y), (¬x ∨ z)]
{}
[x, (¬x ∨ z)]
{}
[¬y, (¬x ∨ z)]
{}
[(¬x ∨ z)]
{x -> t}
[(¬x ∨ z)]
{y -> f}
[¬x]
{x -> t}
[z]
{x -> t}
[]
{x -> t,
z -> t}


(x ∨ ¬y) ∧ (¬x ∨ z)
[(x ∨ ¬y), (¬x ∨ z)]
{}
[x, (¬x ∨ z)]
{}
[¬y, (¬x ∨ z)]
{}
[(¬x ∨ z)]
{x -> t}
[(¬x ∨ z)]
{y -> f}
[¬x]
{x -> t}
[z]
{x -> t}
[¬x]
{y -> f}
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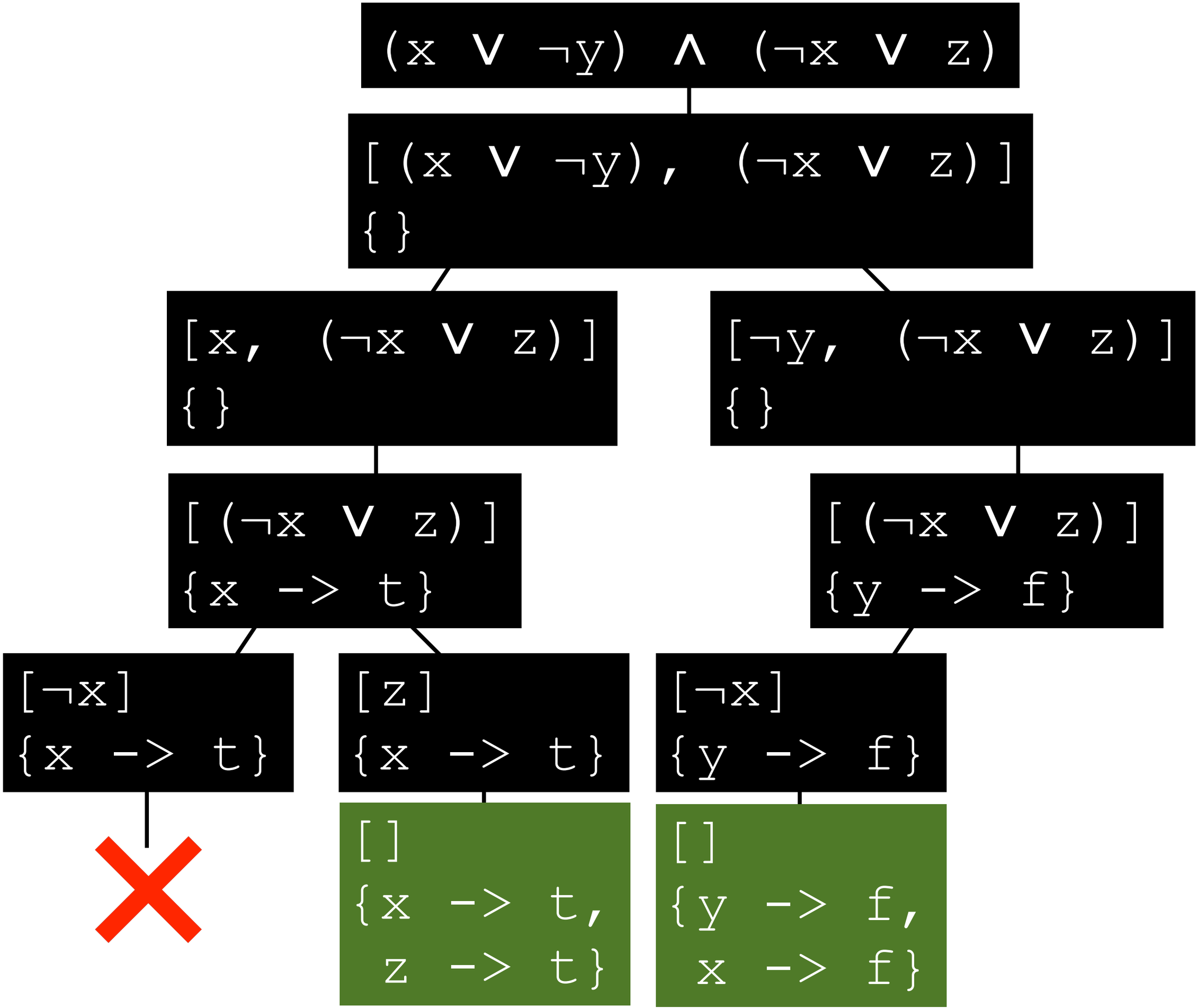
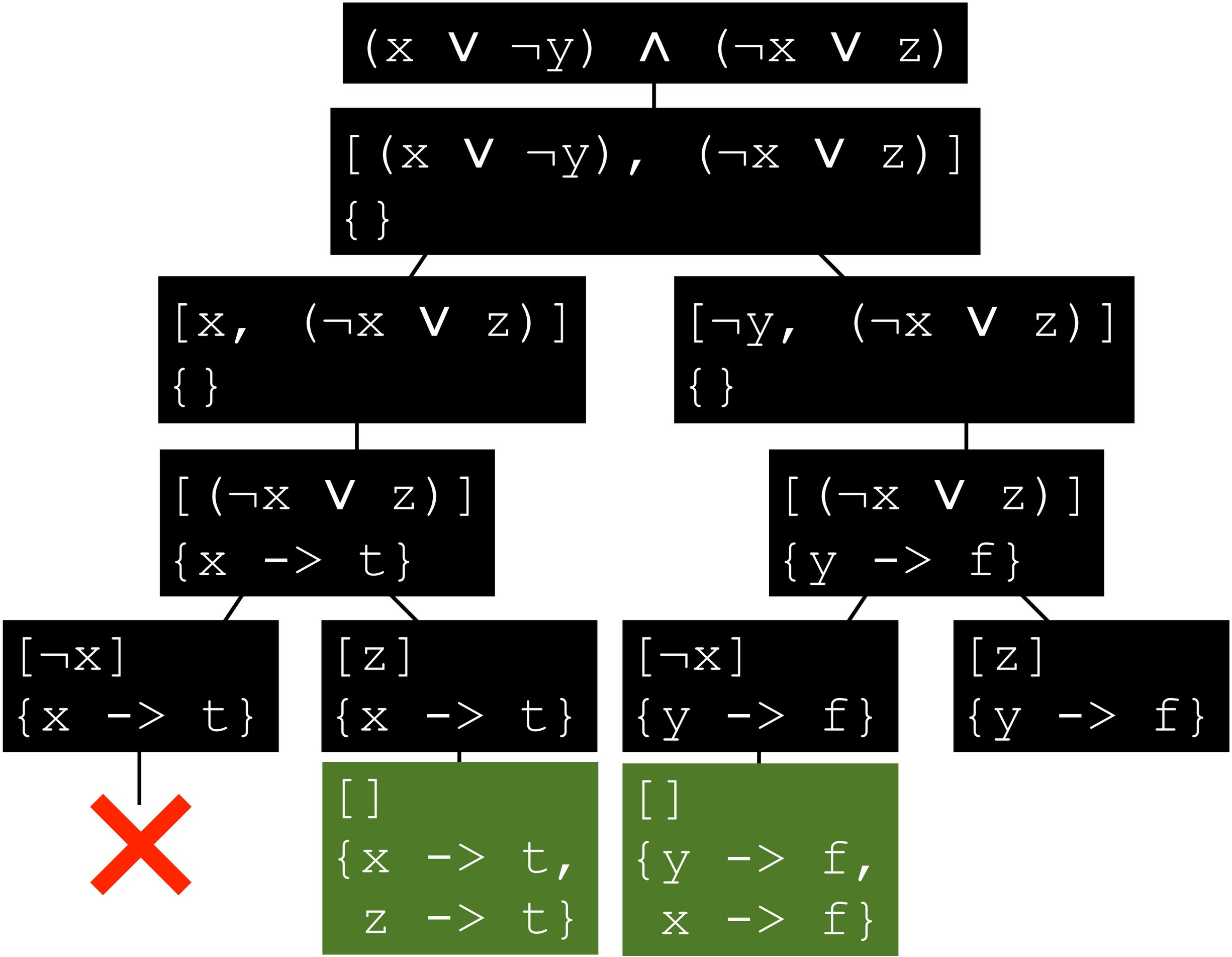

(x ∨ ¬y) ∧ (¬x ∨ z)
[(x ∨ ¬y), (¬x ∨ z)]
{}
[x, (¬x ∨ z)]
{}
[¬y, (¬x ∨ z)]
{}
[(¬x ∨ z)]
{x -> t}
[(¬x ∨ z)]
{y -> f}
[¬x]
{x -> t}
[z]
{x -> t}
[¬x]
{y -> f}
[]
{x -> t,
z -> t}
[]
{y -> f,
x -> f}


(x ∨ ¬y) ∧ (¬x ∨ z)
[(x ∨ ¬y), (¬x ∨ z)]
{}
[x, (¬x ∨ z)]
{}
[¬y, (¬x ∨ z)]
{}
[(¬x ∨ z)]
{x -> t}
[(¬x ∨ z)]
{y -> f}
[¬x]
{x -> t}
[z]
{x -> t}
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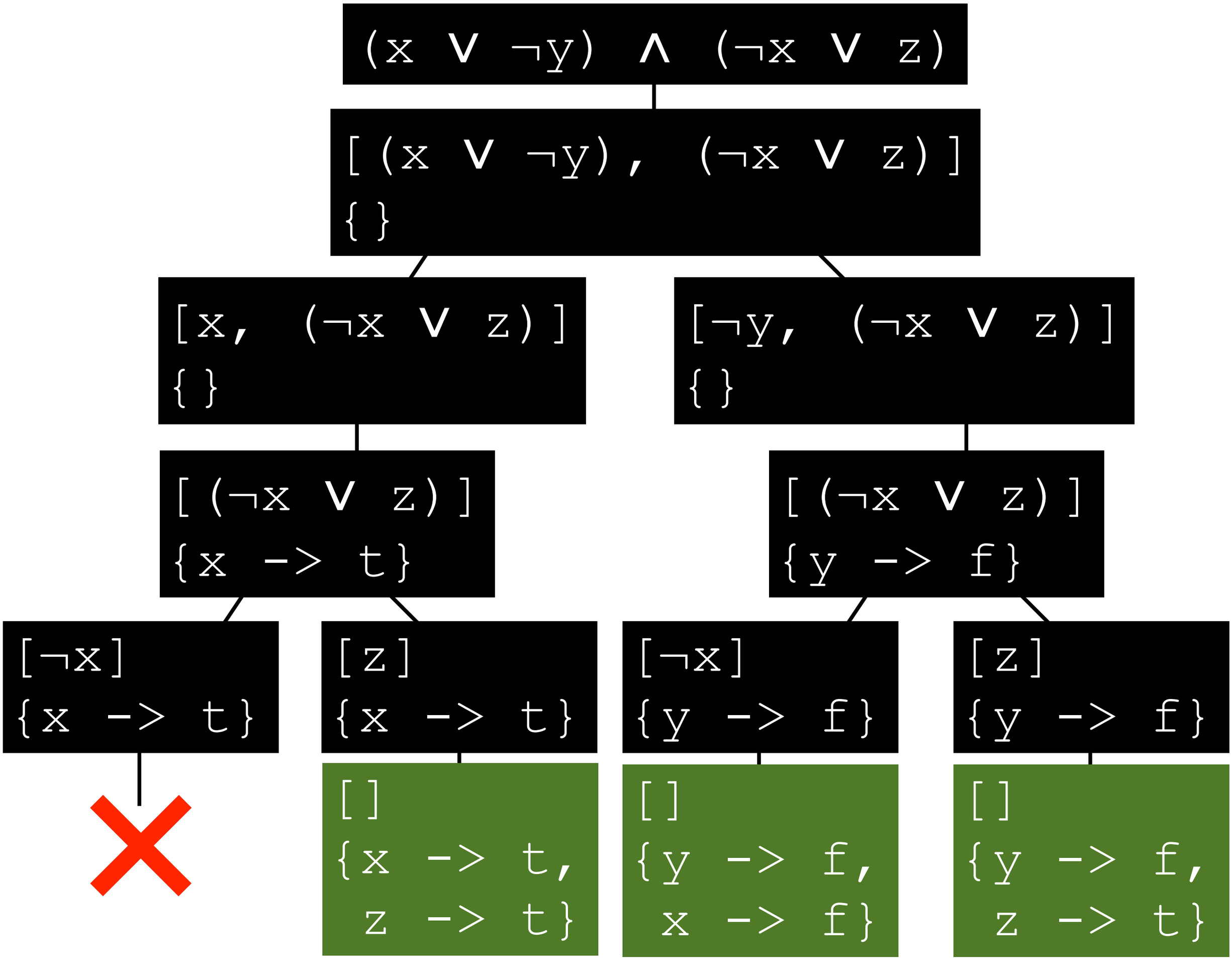
{y -> f}
[z]
{y -> f}
[]
{x -> t,
z -> t}
[]
{y -> f,
x -> f}


(x ∨ ¬y) ∧ (¬x ∨ z)
[(x ∨ ¬y), (¬x ∨ z)]
{}
[x, (¬x ∨ z)]
{}
[¬y, (¬x ∨ z)]
{}
[(¬x ∨ z)]
{x -> t}
[(¬x ∨ z)]
{y -> f}
[¬x]
{x -> t}
[z]
{x -> t}
[¬x]
{y -> f}
[z]
{y -> f}
[]
{x -> t,
z -> t}
[]
{y -> f,
x -> f}
[]
{y -> f,
z -> t}

Exercise: Second Side of SAT Sheet