

COMP 410: SAT and Semantic Tableau

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SAT Background

SAT

- Short for the Boolean satisfiability problem
- Given a Boolean formula with variables, is there an assignment of true/false to the variables which makes the formula true?

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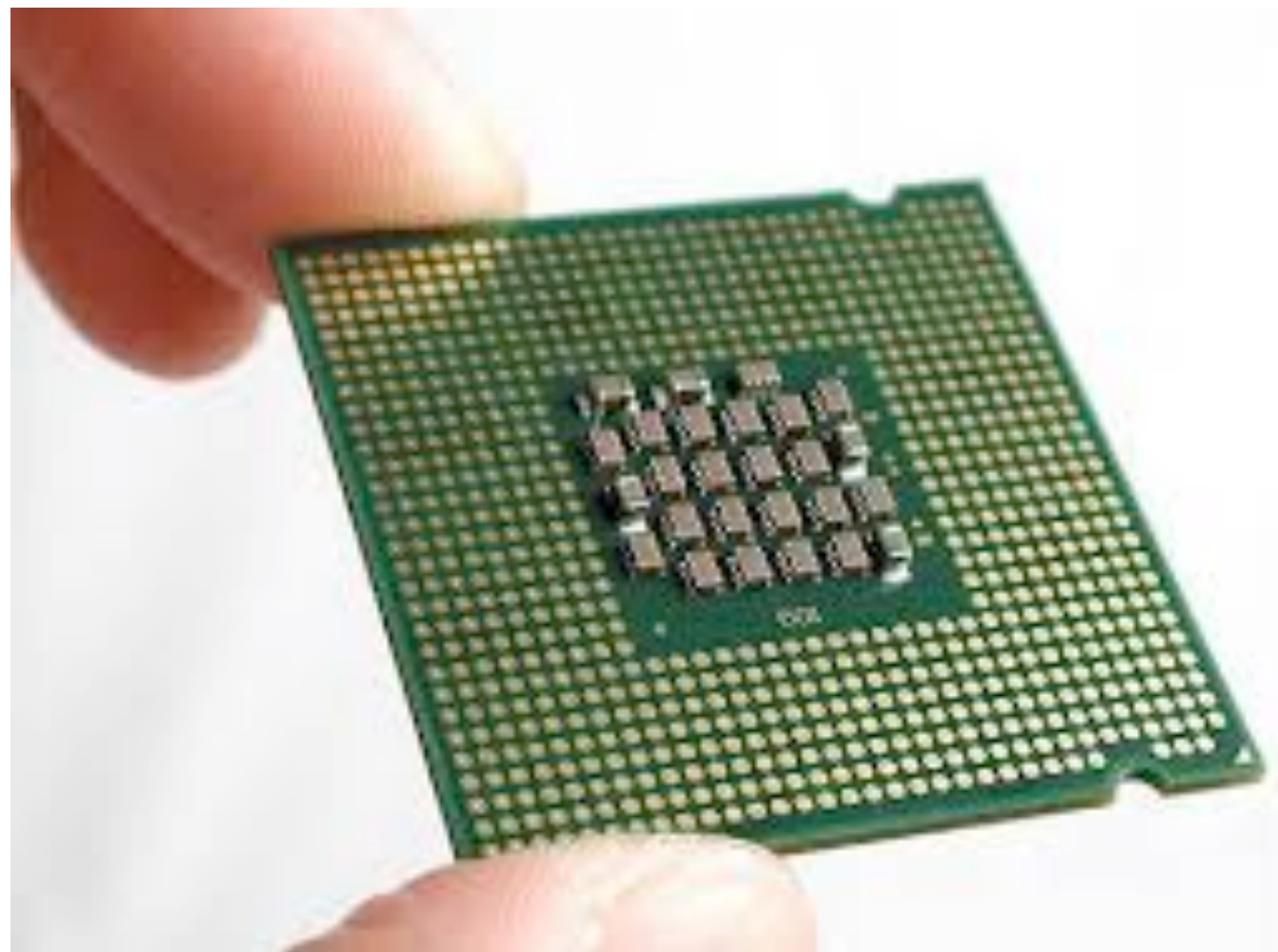
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Relevance

Widespread usage in hardware and software verification

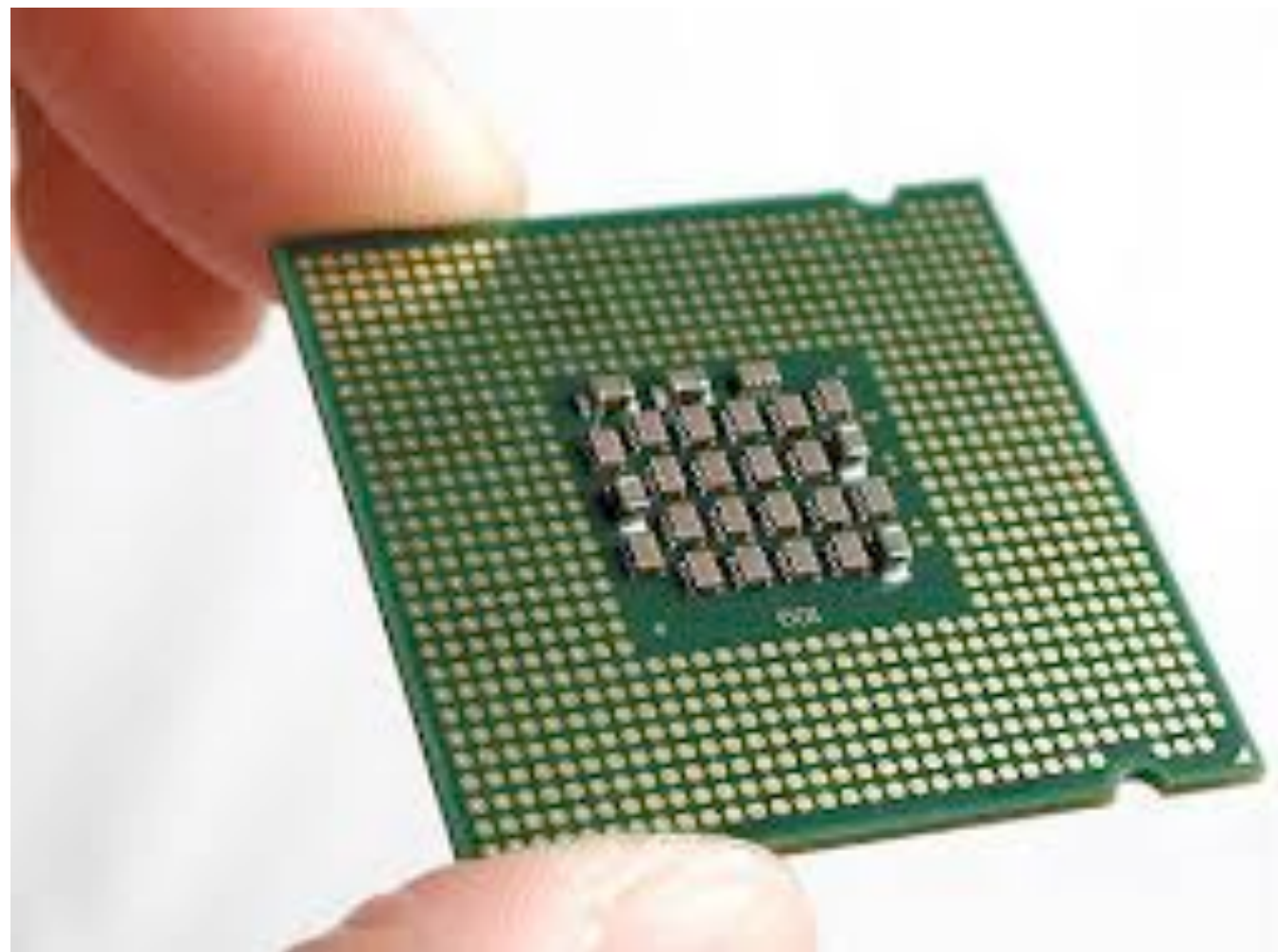
Relevance

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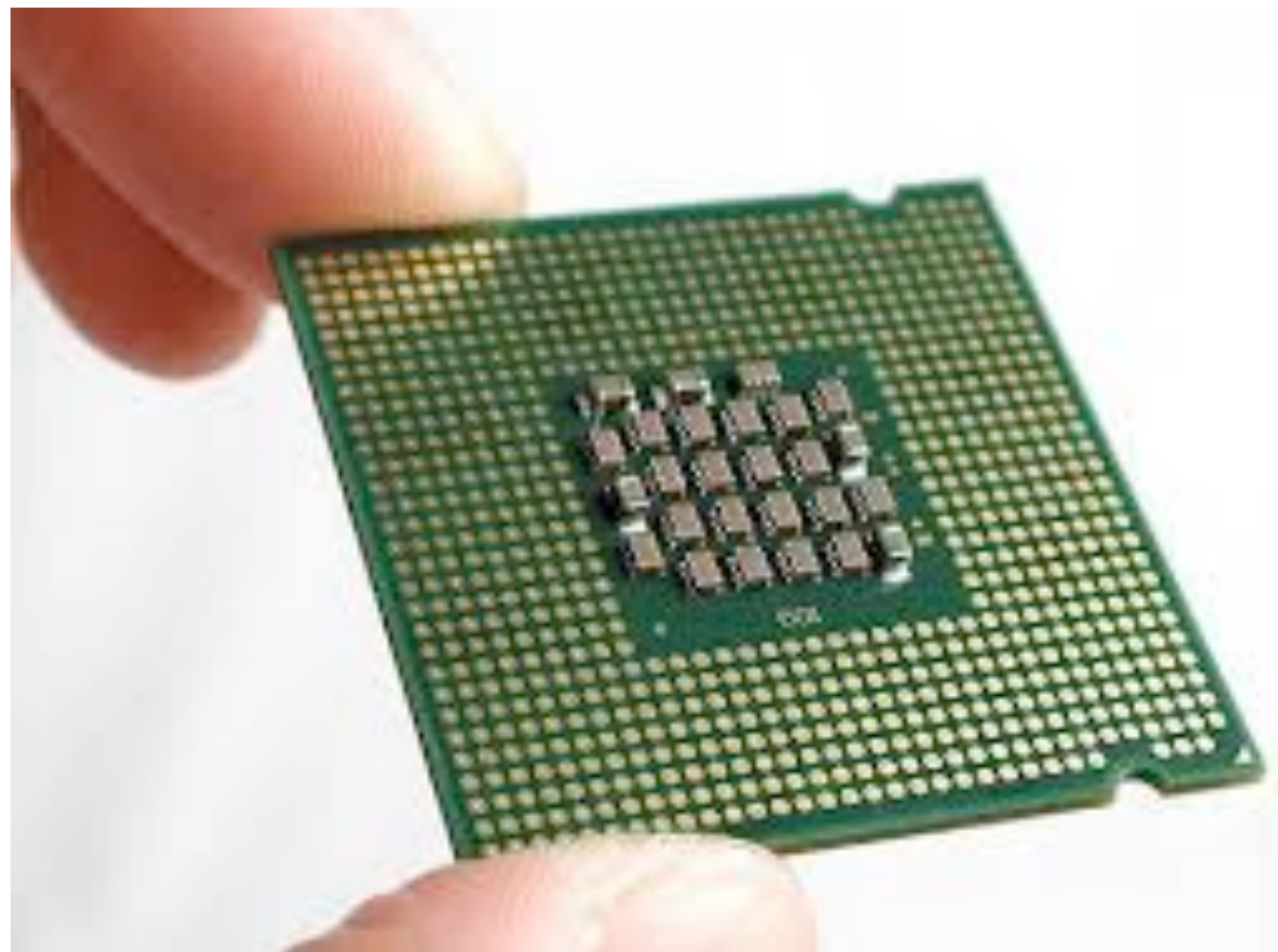
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Relevance to Logic Programming

- Methods for solving SAT can be used to execute logic programs
- Logic programming can be phrased as SAT with some additional stuff

Semantic Tableau

- One method for solving SAT instances
- Basic idea: iterate over the formula
 - Maintain subformulas that must be true
 - Learn which variables must be true/false
 - Stop at conflicts (unsatisfiable), or when no subformulas remain (have solution)

Positive Literals

a

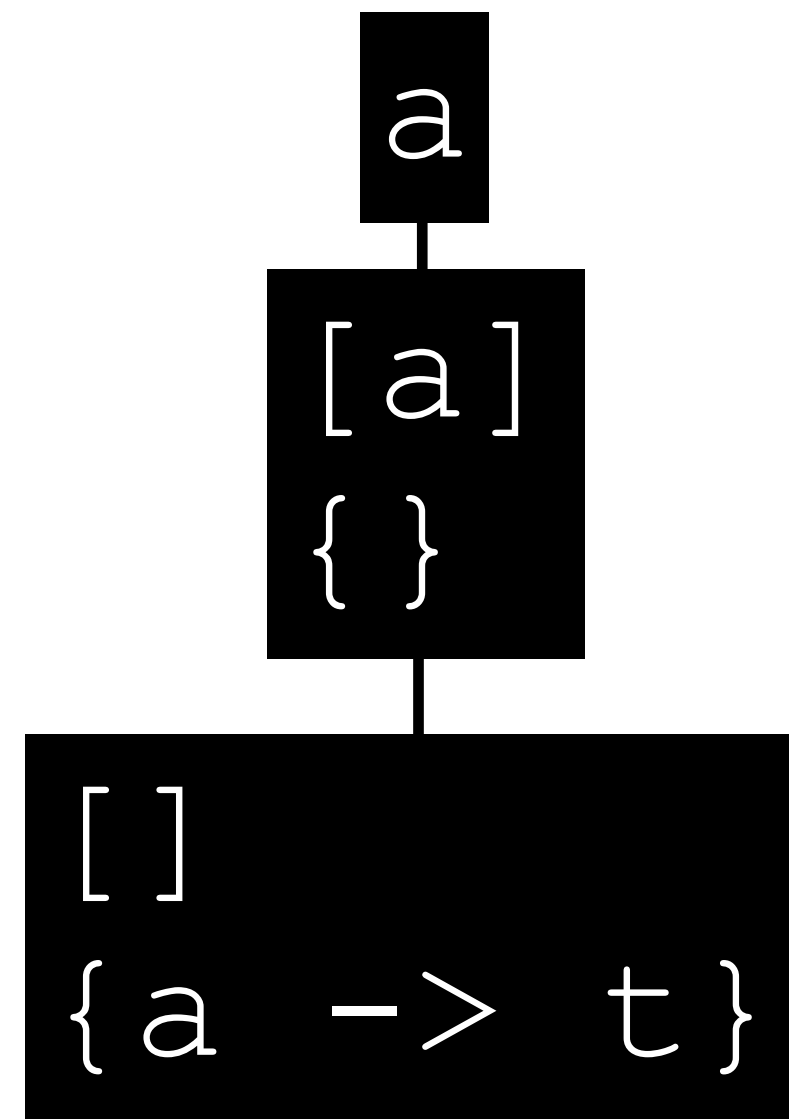
Positive Literals

a

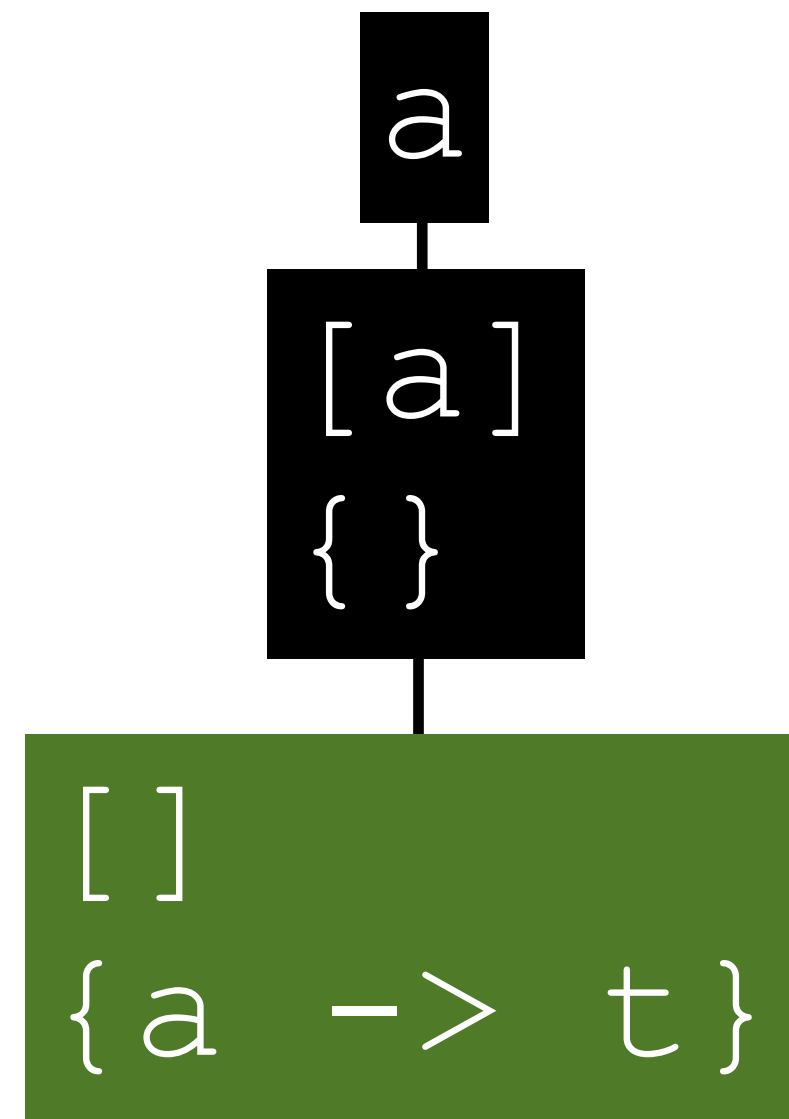
[a]

{ }

Positive Literals



Positive Literals



Negative Literals

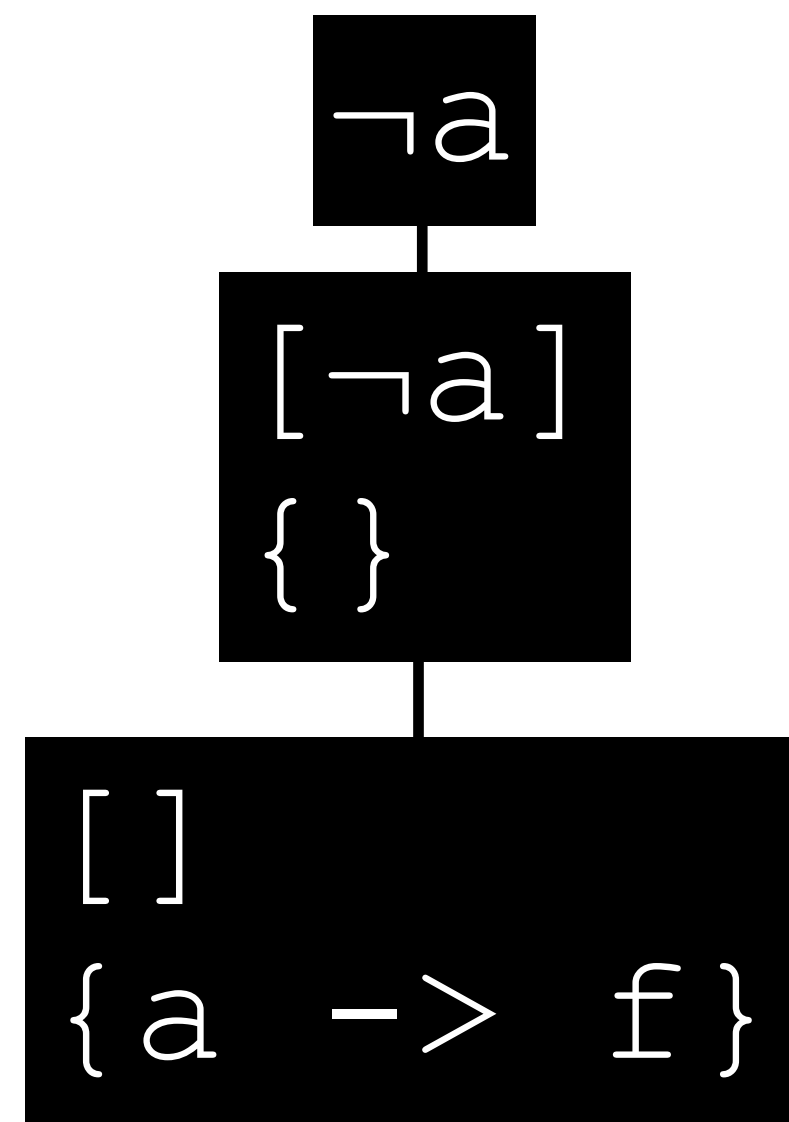
$$\neg a$$

Negative Literals

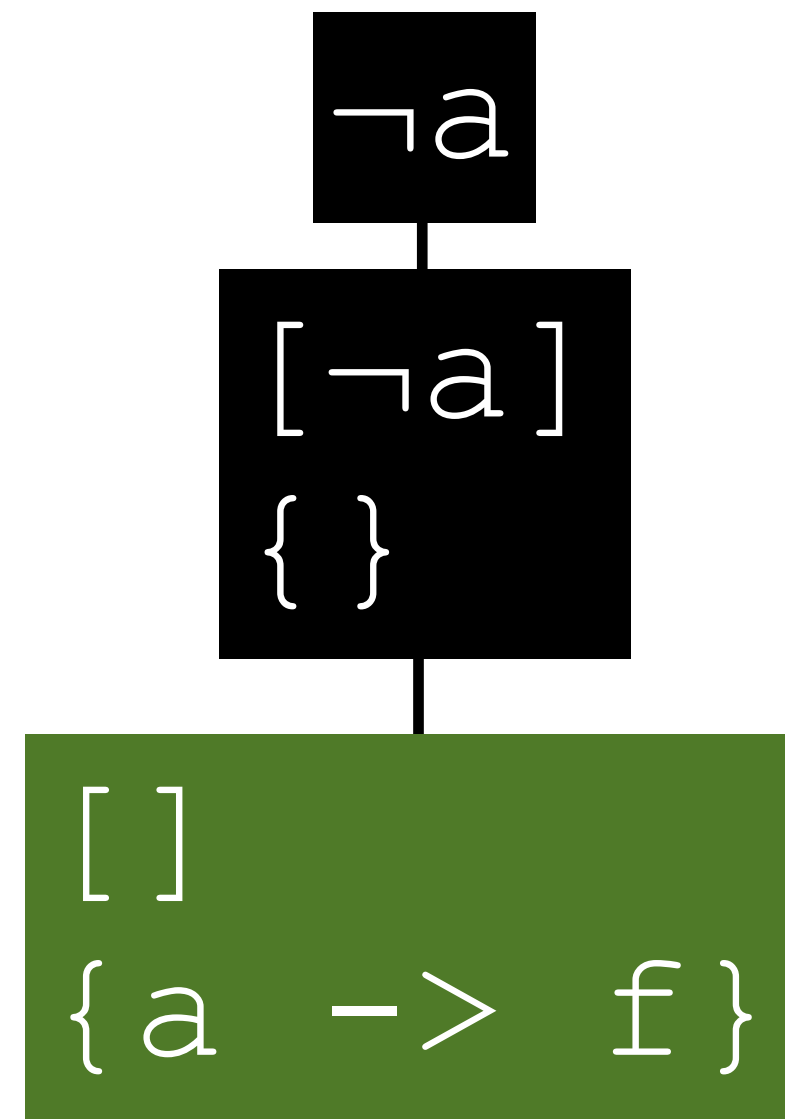
$\neg a$

$[\neg a]$
 $\{\}$

Negative Literals



Negative Literals



Logical And

$a \wedge b$

Logical And

$a \wedge b$

$[a \wedge b]$
 $\{\}$

Logical And

$a \wedge b$

$[a \wedge b]$
 $\{\}$

$[a, b]$
 $\{\}$

Logical And

$a \wedge b$

$[a \wedge b]$
 $\{\}$

$[a, b]$
 $\{\}$

$[b]$
 $\{a \rightarrow t\}$

Logical And

$a \wedge b$

$[a \wedge b]$
 $\{\}$

$[a, b]$
 $\{\}$

$[b]$
 $\{a \rightarrow t\}$

$[]$
 $\{a \rightarrow t, b \rightarrow t\}$

Logical And

$a \wedge b$

$[a \wedge b]$
 $\{\}$

$[a, b]$
 $\{\}$

$[b]$
 $\{a \rightarrow t\}$

$[]$
 $\{a \rightarrow t, b \rightarrow t\}$

Logical And

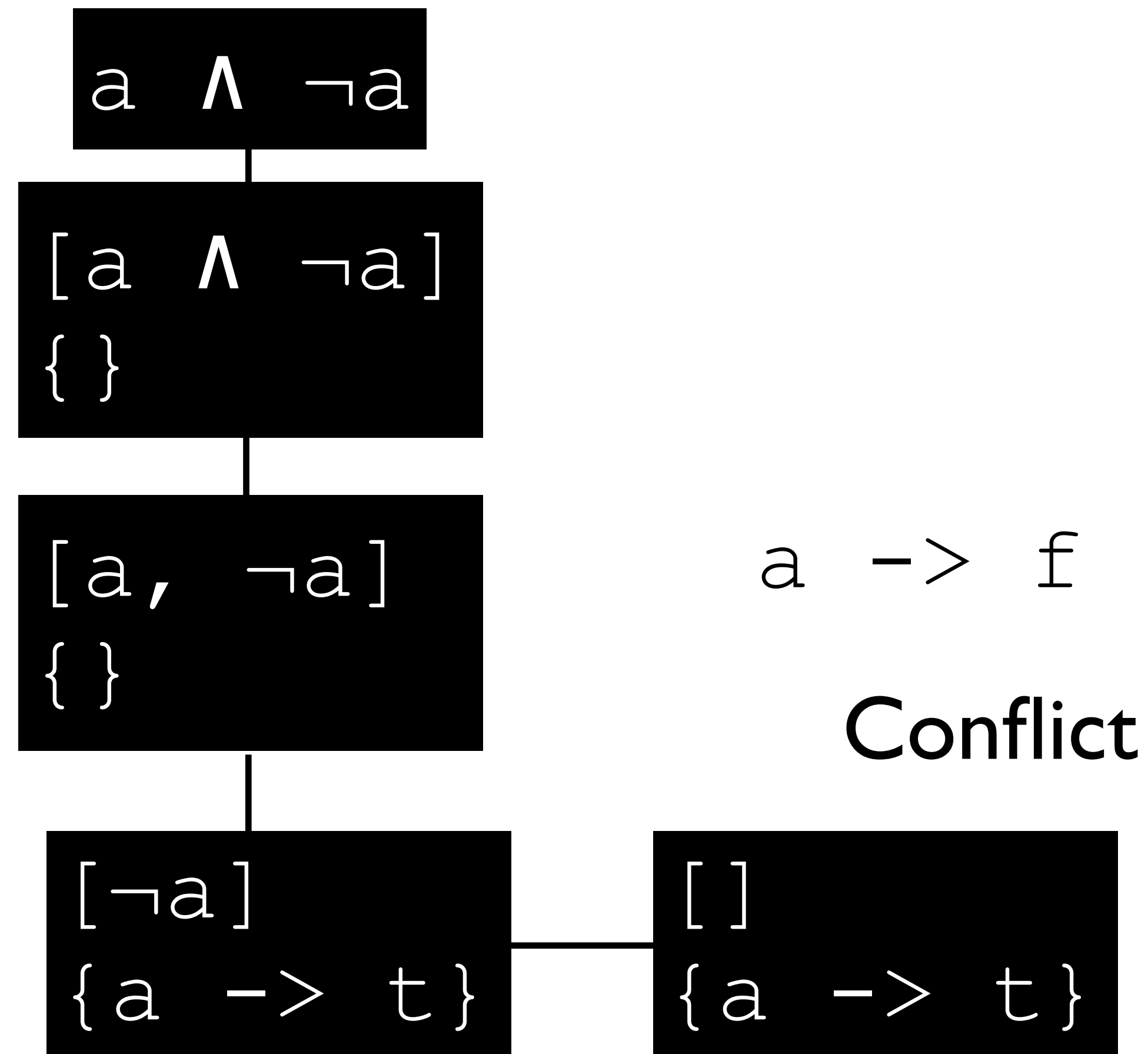
$a \wedge \neg a$

Logical And

$a \wedge \neg a$

$[a \wedge \neg a]$
 $\{\}$

Logical And

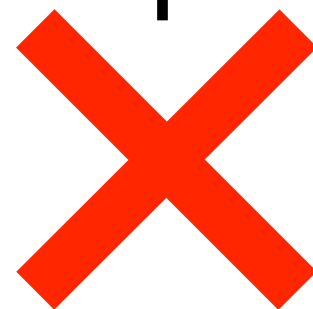


Logical And

$a \wedge \neg a$

$[a \wedge \neg a]$
 $\{\}$

$[\neg a]$
 $\{a \rightarrow t\}$



Exercise: First Side of SAT Sheet

Logical Or

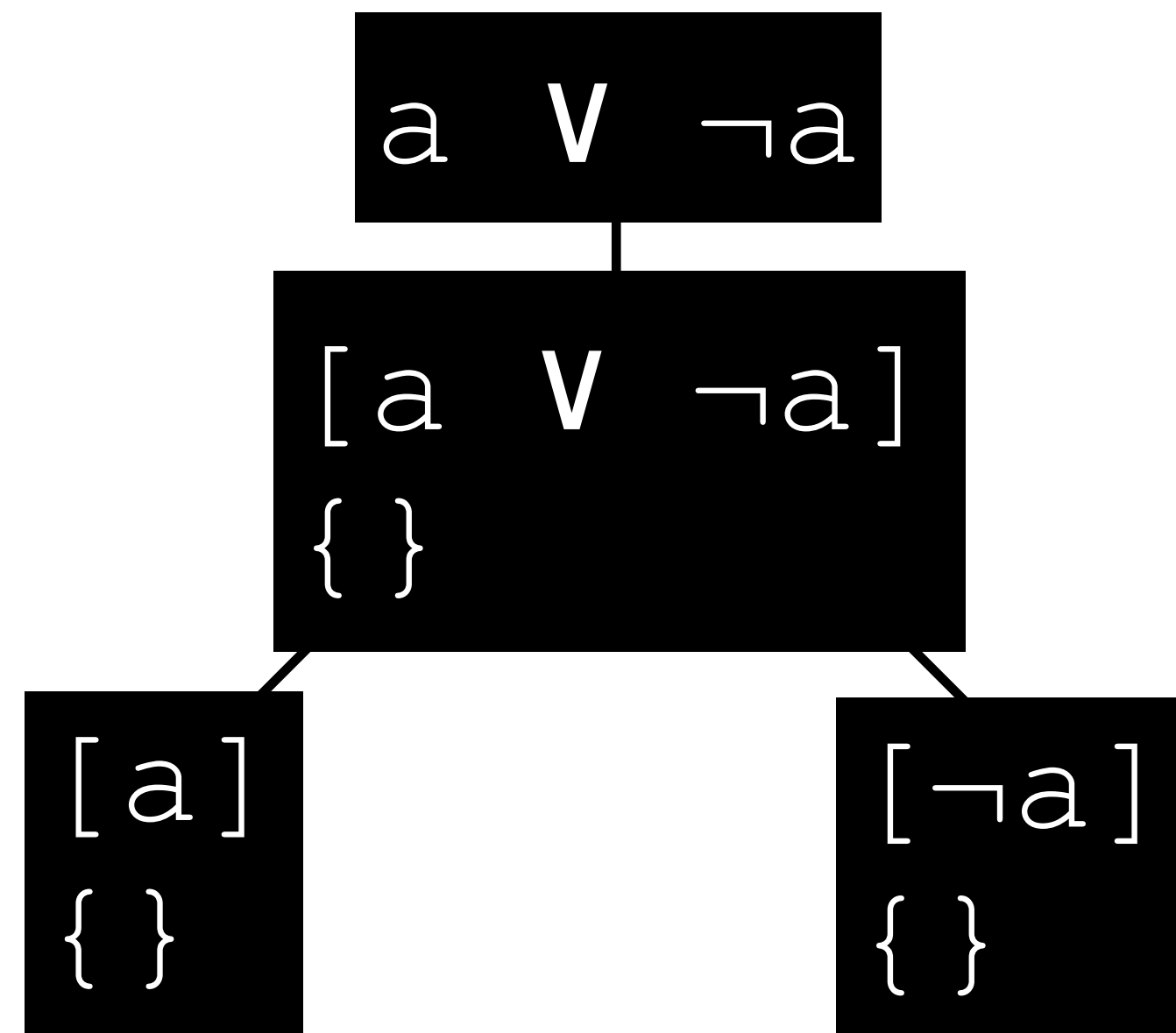
$$a \vee \neg a$$

Logical Or

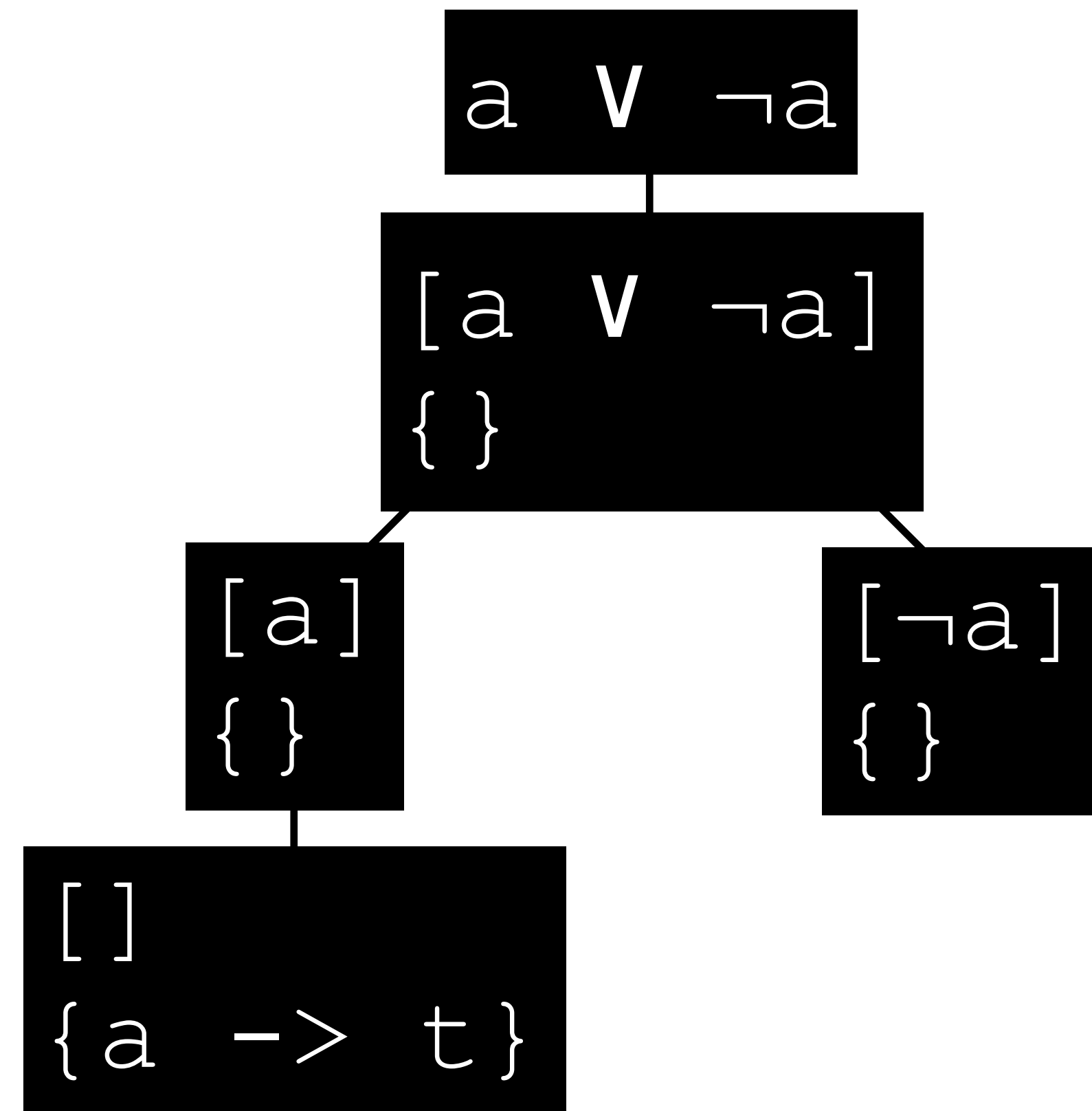
$a \vee \neg a$

$[a \vee \neg a]$
 $\{\}$

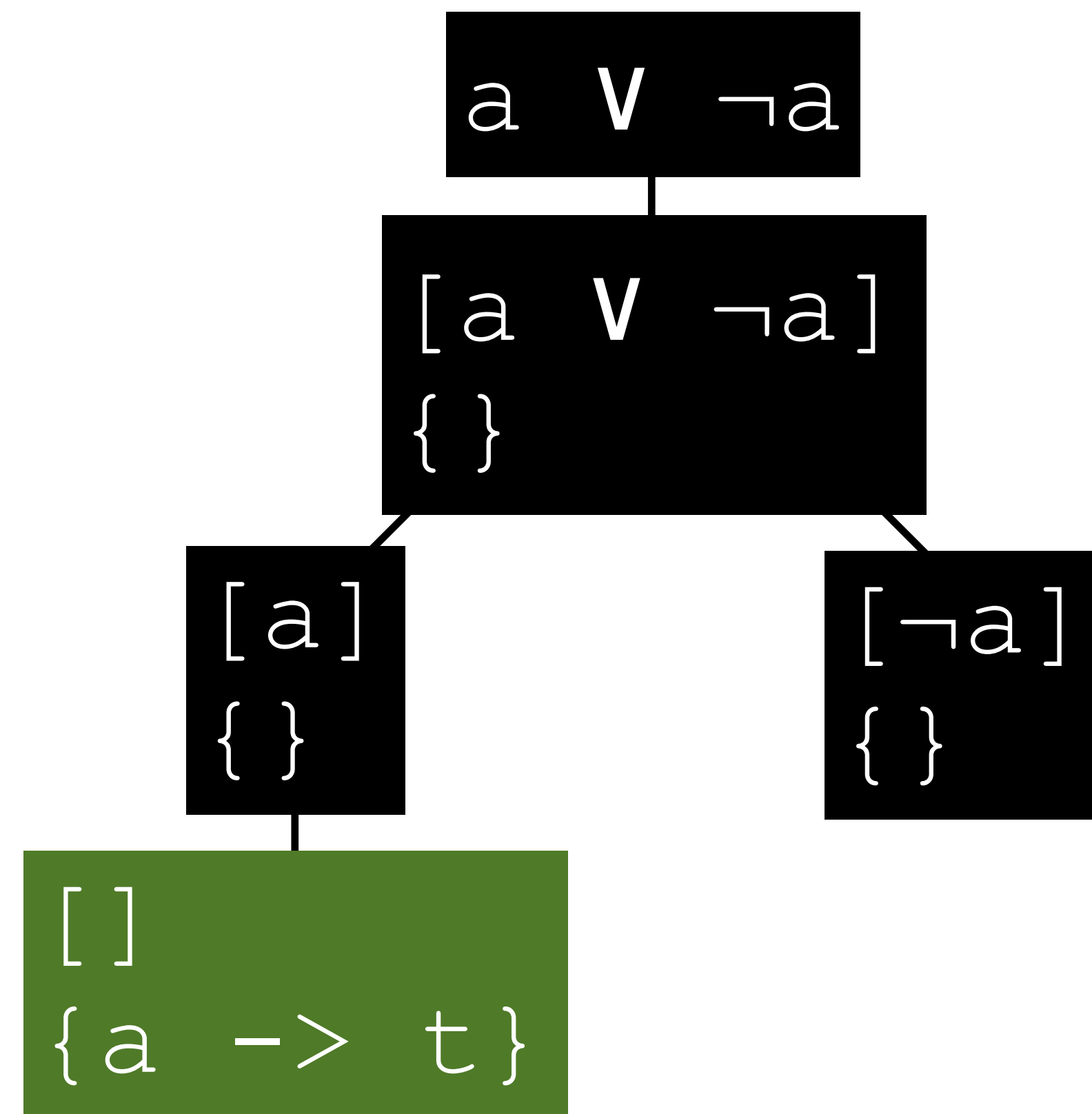
Logical Or



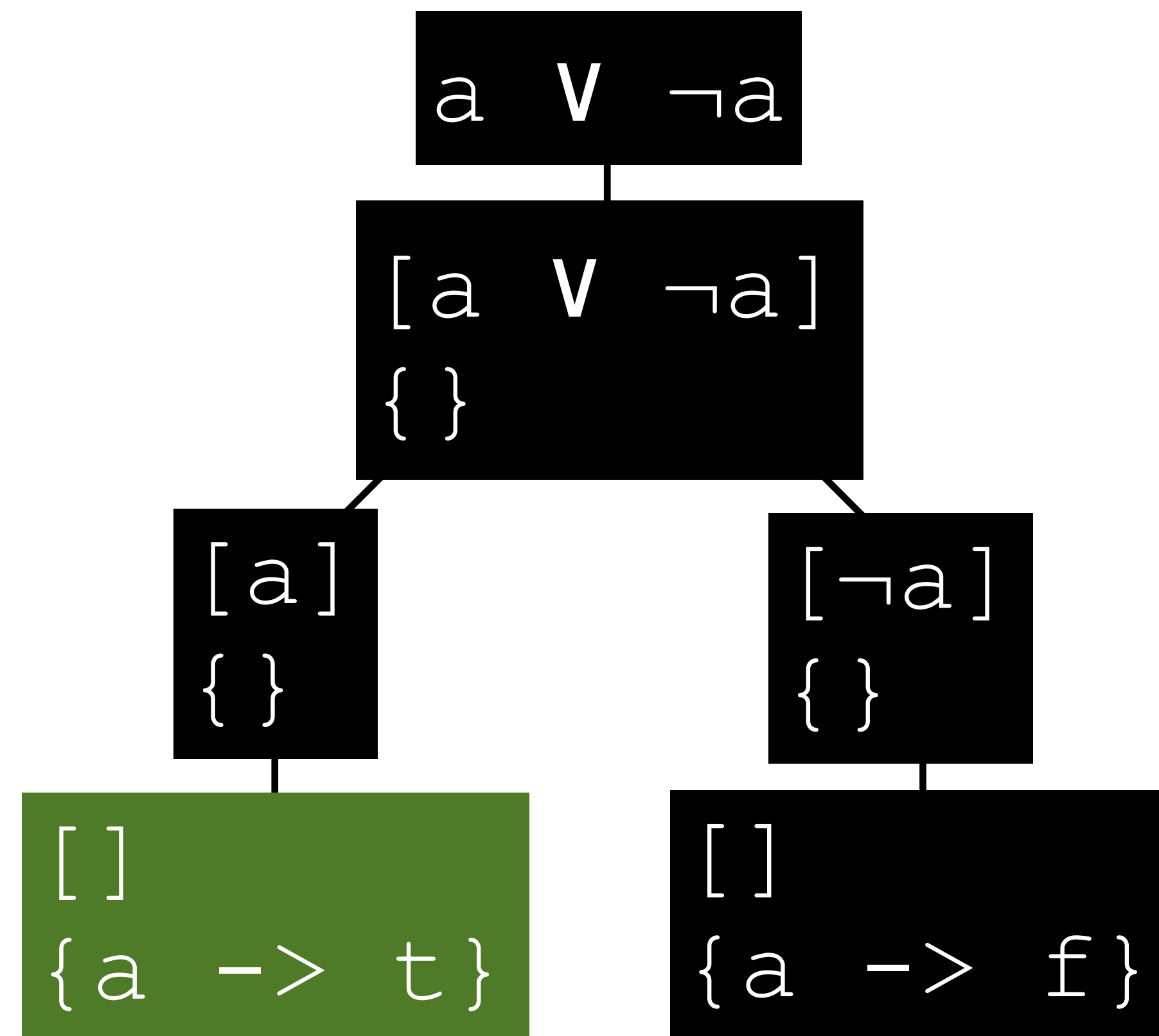
Logical Or



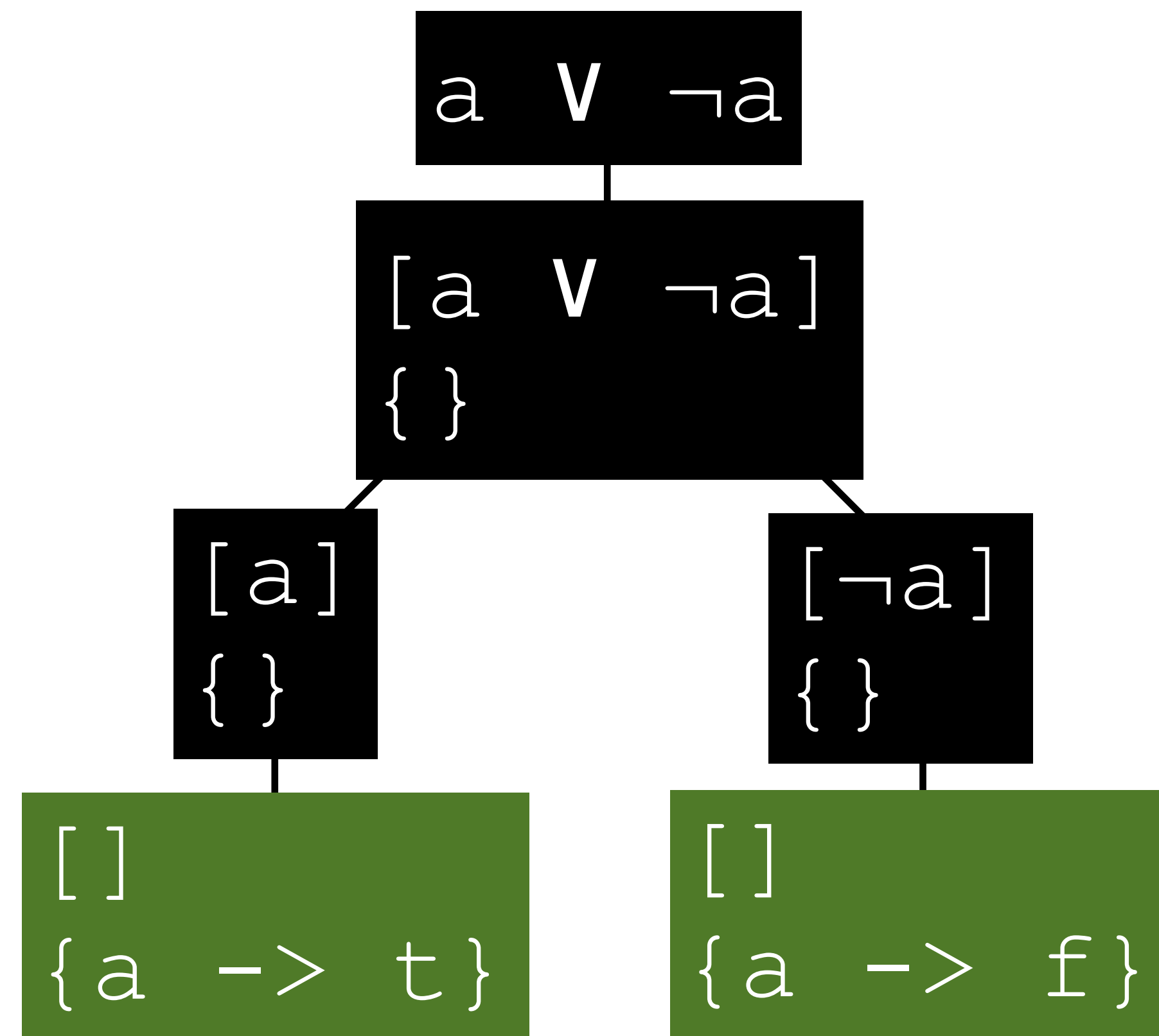
Logical Or



Logical Or



Logical Or



Examples

Example 1:

$$(\neg b \vee a) \wedge b$$

$$(\neg b \vee a) \wedge b$$

$$(\neg b \vee a) \wedge b$$

$$\begin{array}{l} [(\neg b \vee a), b] \\ \{\} \end{array}$$

$(\neg b \vee a) \wedge b$

$[(\neg b \vee a), b]$
 $\{\}$

$[\neg b, b]$
 $\{\}$

$(\neg b \vee a) \wedge b$

$[(\neg b \vee a), b]$
 $\{\}$

$[\neg b, b]$
 $\{\}$

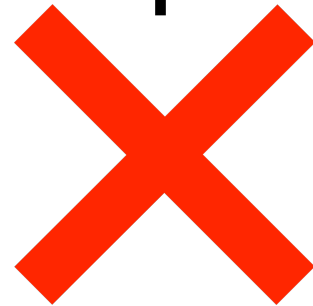
$[b]$
 $\{b \rightarrow f\}$

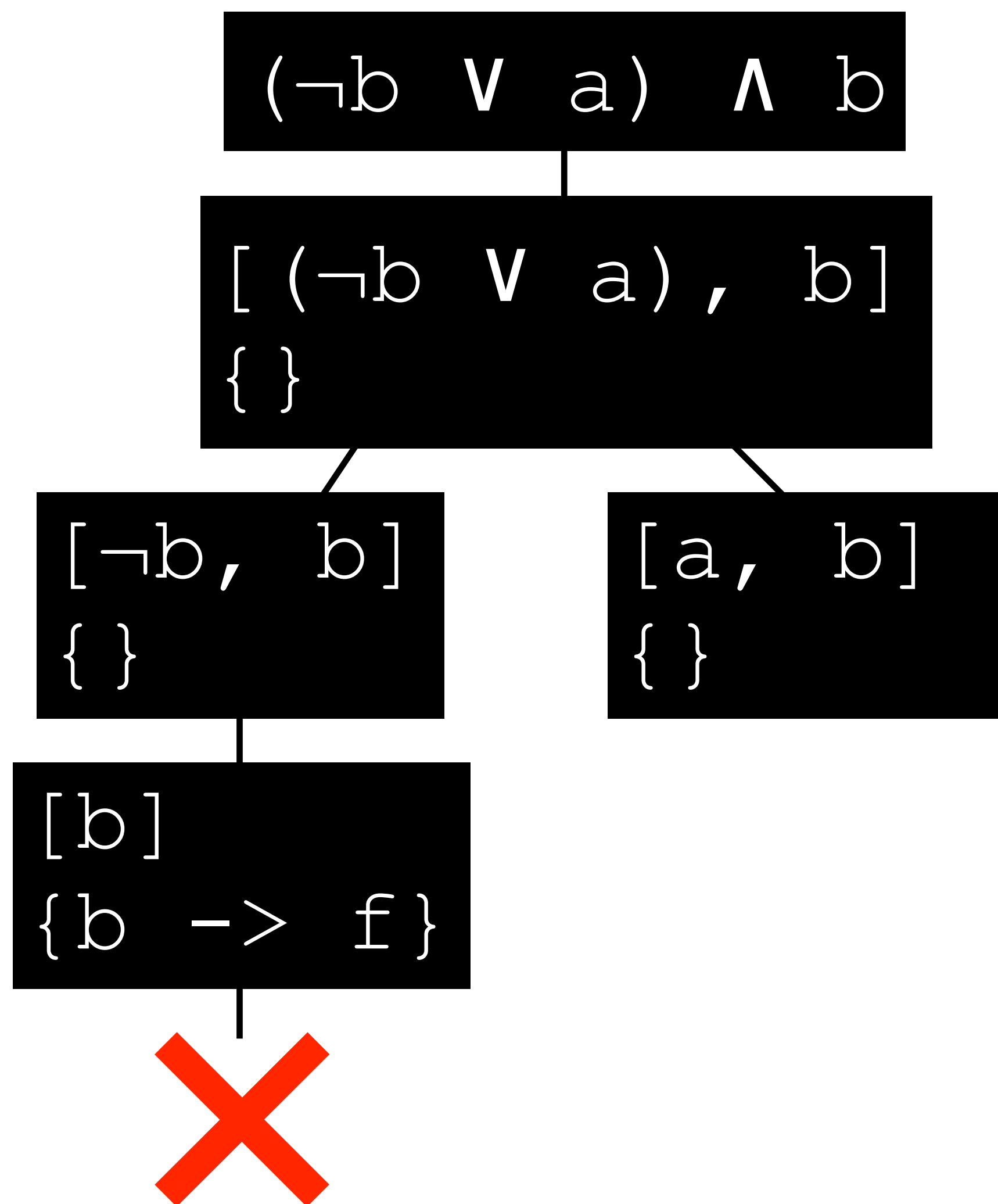
$(\neg b \vee a) \wedge b$

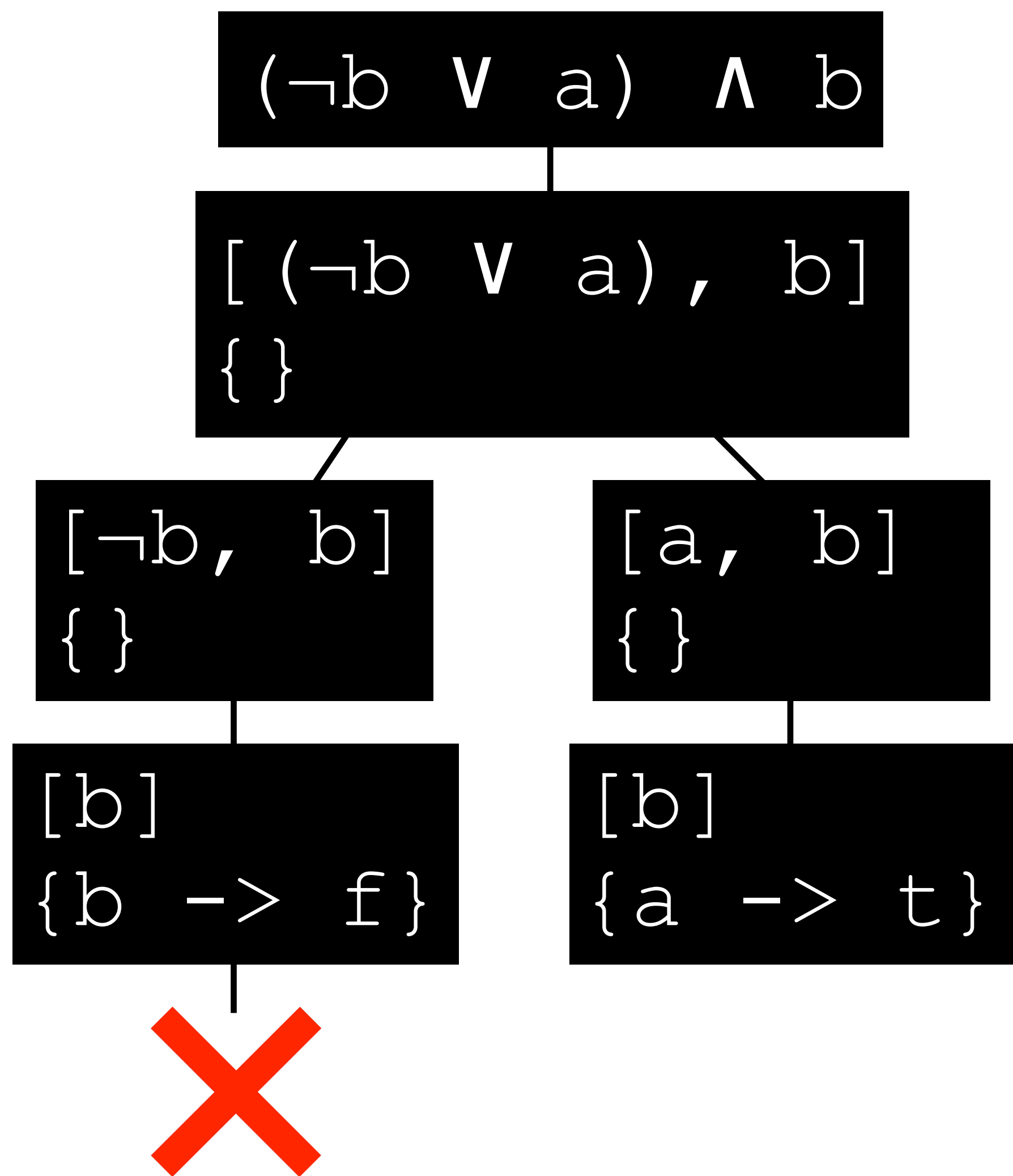
$[(\neg b \vee a), b]$
 $\{\}$

$[\neg b, b]$
 $\{\}$

$[b]$
 $\{b \rightarrow f\}$







$(\neg b \vee a) \wedge b$

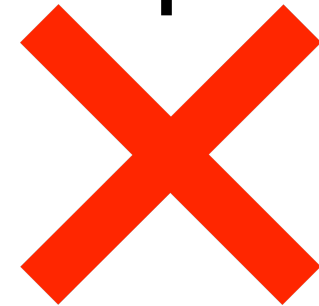
$[(\neg b \vee a), b]$
 $\{\}$

$[\neg b, b]$
 $\{\}$

$[a, b]$
 $\{\}$

$[b]$
 $\{b \rightarrow f\}$

$[b]$
 $\{a \rightarrow t\}$



$[]$
 $\{a \rightarrow t,$
 $b \rightarrow t\}$

Example 2:

$$(x \vee \neg y) \wedge (\neg x \vee z)$$

$$(x \vee \neg y) \wedge (\neg x \vee z)$$

$$(x \vee \neg y) \wedge (\neg x \vee z)$$
$$[(x \vee \neg y), (\neg x \vee z)]$$
$$\{ \}$$

$(x \vee \neg y) \wedge (\neg x \vee z)$

$[(x \vee \neg y), (\neg x \vee z)]$
 $\{ \}$

$[x, (\neg x \vee z)]$
 $\{ \}$

$(x \vee \neg y) \wedge (\neg x \vee z)$

$[(x \vee \neg y), (\neg x \vee z)]$
 $\{\}$

$[x, (\neg x \vee z)]$
 $\{\}$

$[(\neg x \vee z)]$
 $\{x \rightarrow t\}$

$(x \vee \neg y) \wedge (\neg x \vee z)$

$[(x \vee \neg y), (\neg x \vee z)]$
 $\{\}$

$[x, (\neg x \vee z)]$
 $\{\}$

$[(\neg x \vee z)]$
 $\{x \rightarrow t\}$

$[\neg x]$
 $\{x \rightarrow t\}$

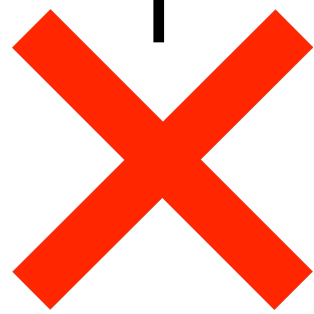
$(x \vee \neg y) \wedge (\neg x \vee z)$

$[(x \vee \neg y), (\neg x \vee z)]$
 $\{\}$

$[x, (\neg x \vee z)]$
 $\{\}$

$[(\neg x \vee z)]$
 $\{x \rightarrow t\}$

$[\neg x]$
 $\{x \rightarrow t\}$



$(x \vee \neg y) \wedge (\neg x \vee z)$

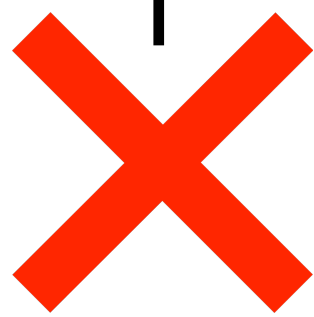
$[(x \vee \neg y), (\neg x \vee z)]$
 $\{\}$

$[x, (\neg x \vee z)]$
 $\{\}$

$[(\neg x \vee z)]$
 $\{x \rightarrow t\}$

$[\neg x]$
 $\{x \rightarrow t\}$

$[z]$
 $\{x \rightarrow t\}$



$(x \vee \neg y) \wedge (\neg x \vee z)$

$[(x \vee \neg y), (\neg x \vee z)]$
 $\{\}$

$[x, (\neg x \vee z)]$
 $\{\}$

$[(\neg x \vee z)]$
 $\{x \rightarrow t\}$

$[\neg x]$
 $\{x \rightarrow t\}$



$[z]$
 $\{x \rightarrow t\}$

$[\]$
 $\{x \rightarrow t,$
 $z \rightarrow t\}$

$(x \vee \neg y) \wedge (\neg x \vee z)$

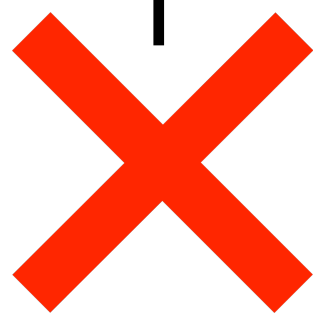
$[(x \vee \neg y), (\neg x \vee z)]$
 $\{\}$

$[x, (\neg x \vee z)]$
 $\{\}$

$[\neg y, (\neg x \vee z)]$
 $\{\}$

$[(\neg x \vee z)]$
 $\{x \rightarrow t\}$

$[\neg x]$
 $\{x \rightarrow t\}$



$[z]$
 $\{x \rightarrow t\}$

$[]$
 $\{x \rightarrow t,$
 $z \rightarrow t\}$

$(x \vee \neg y) \wedge (\neg x \vee z)$

$[(x \vee \neg y), (\neg x \vee z)]$
 $\{\}$

$[x, (\neg x \vee z)]$
 $\{\}$

$[\neg y, (\neg x \vee z)]$
 $\{\}$

$[(\neg x \vee z)]$
 $\{x \rightarrow t\}$

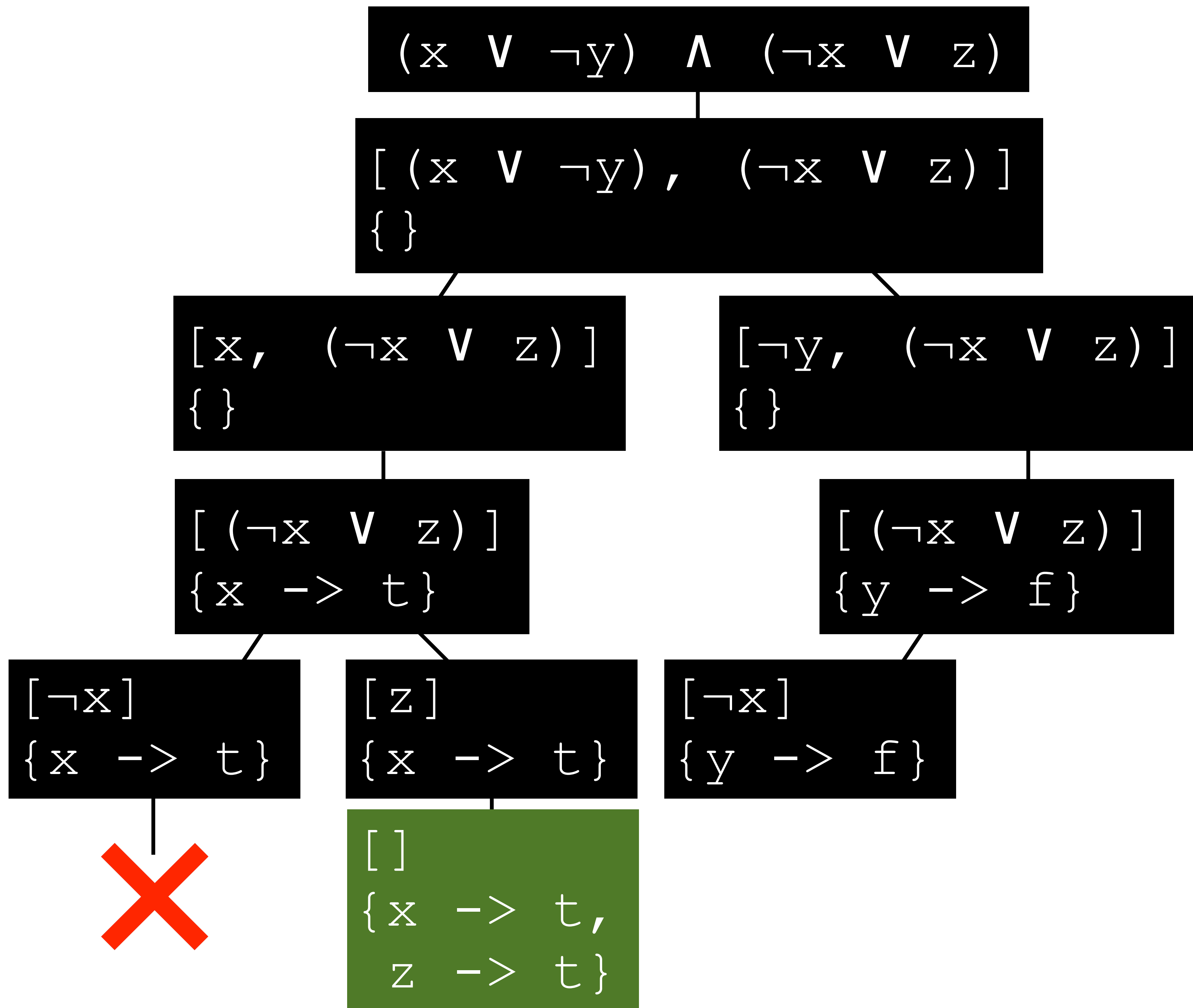
$[(\neg x \vee z)]$
 $\{y \rightarrow f\}$

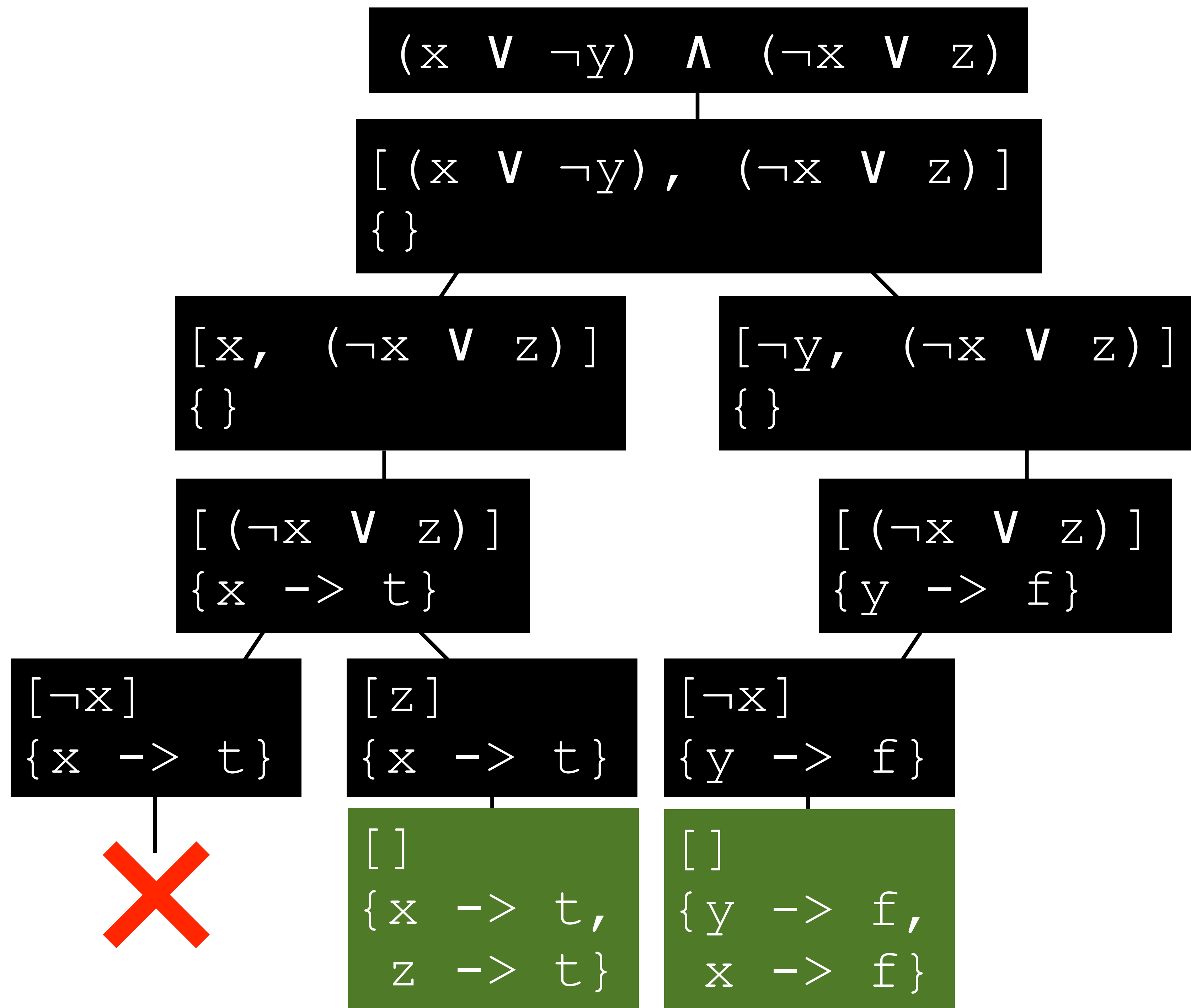
$[\neg x]$
 $\{x \rightarrow t\}$

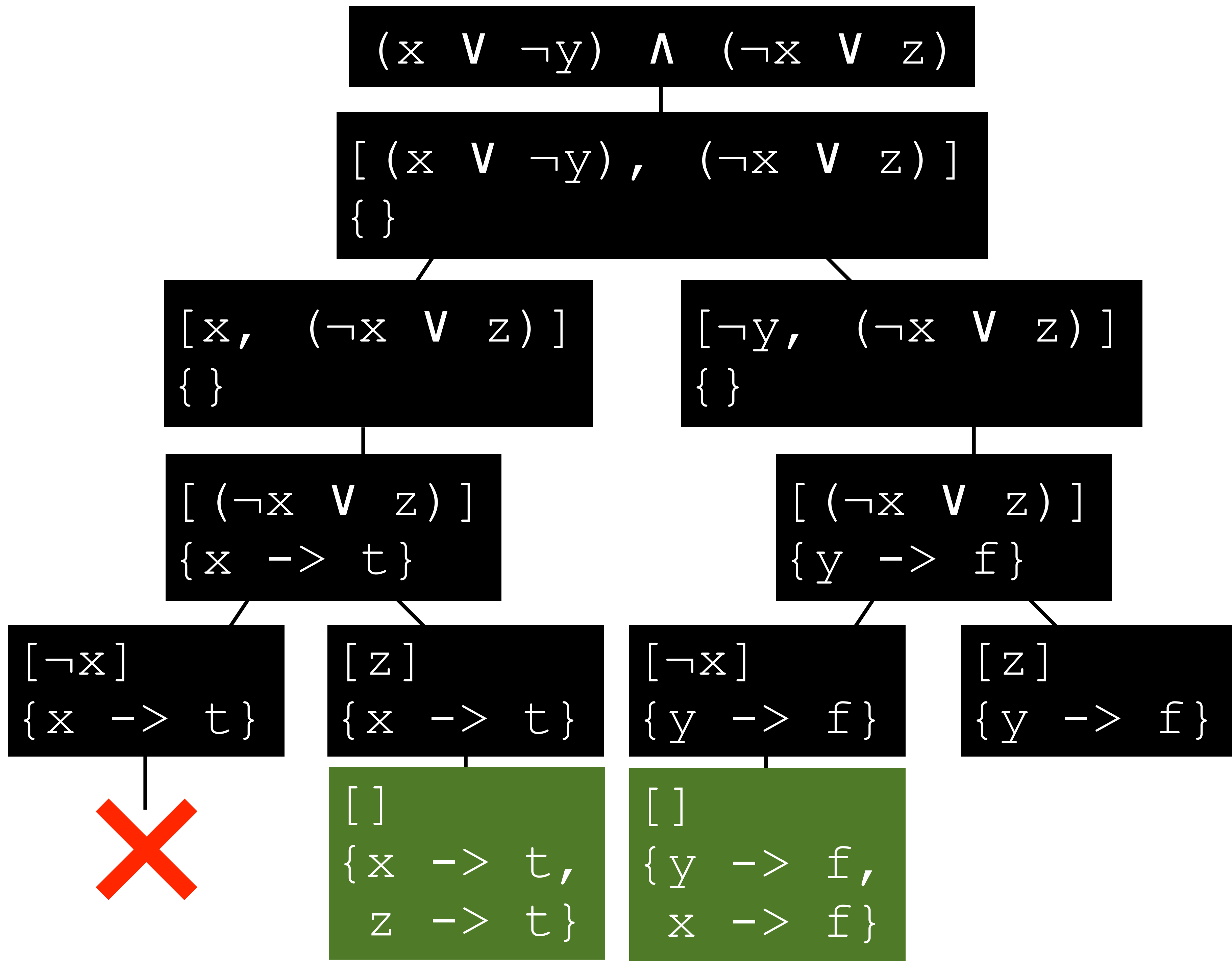
$[z]$
 $\{x \rightarrow t\}$

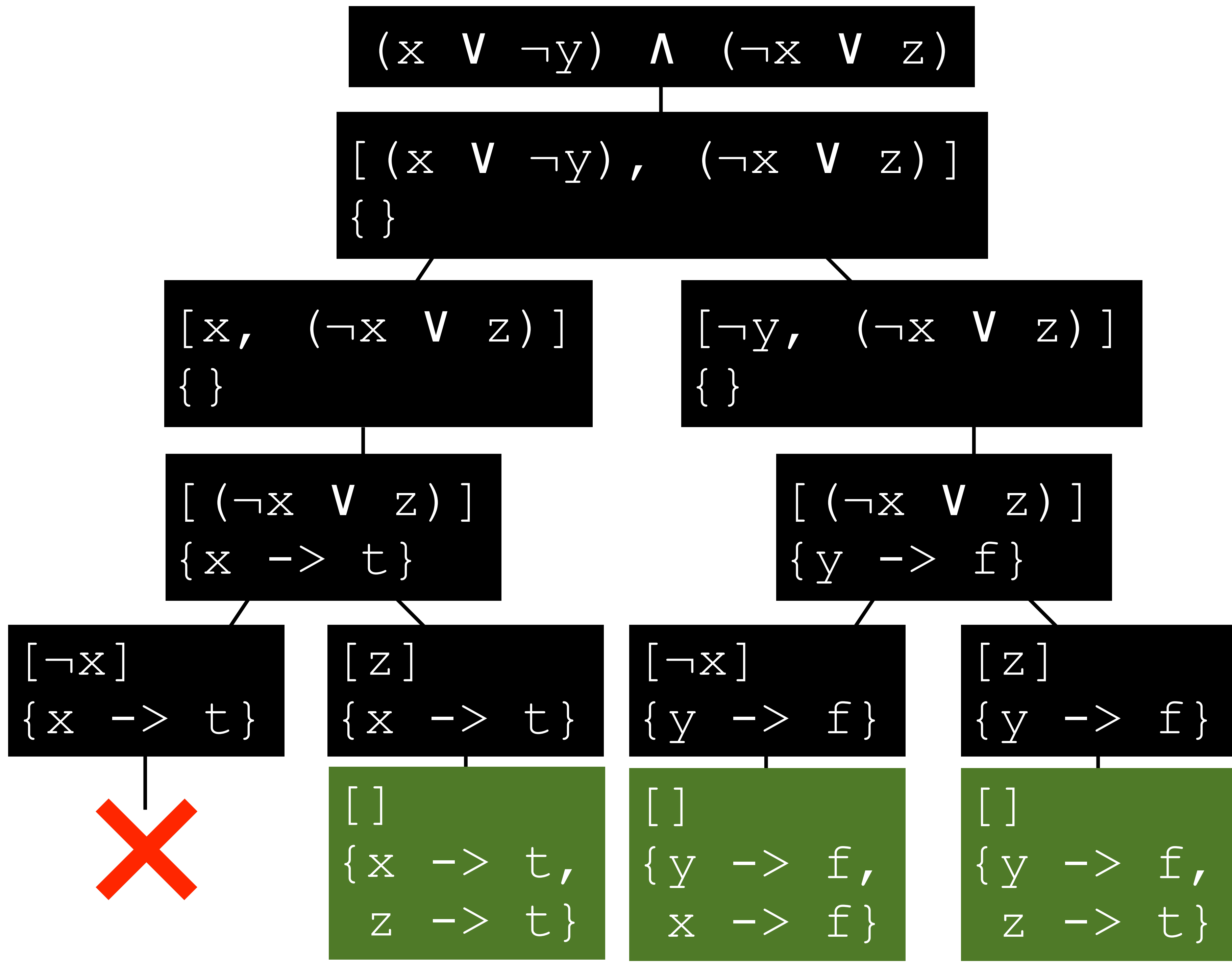


$[]$
 $\{x \rightarrow t,$
 $z \rightarrow t\}$









Exercise: Second Side of SAT Sheet