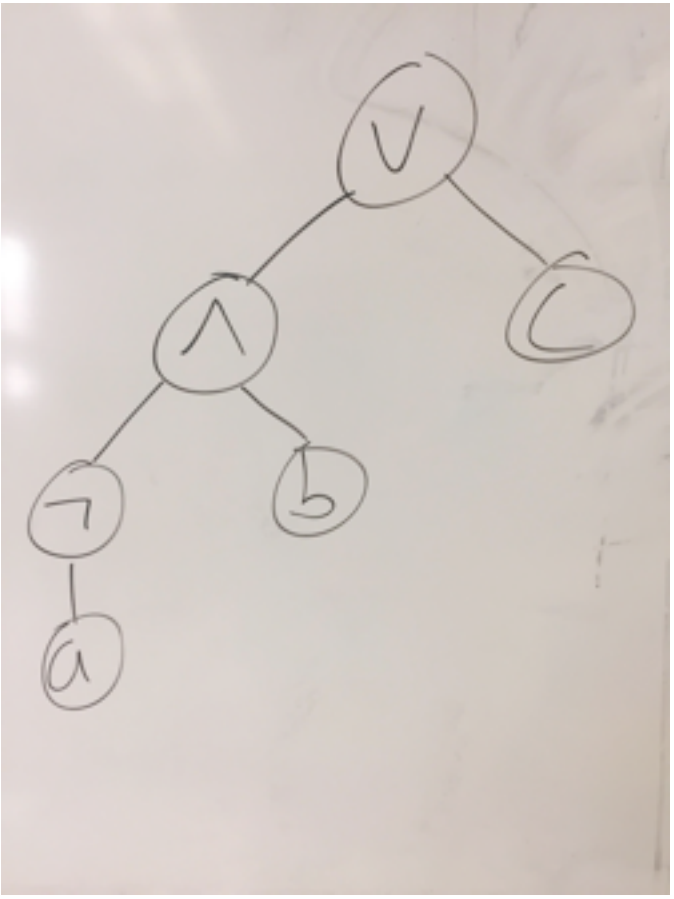
COMP 410
Spring 2018
Midterm Practice Exam #1

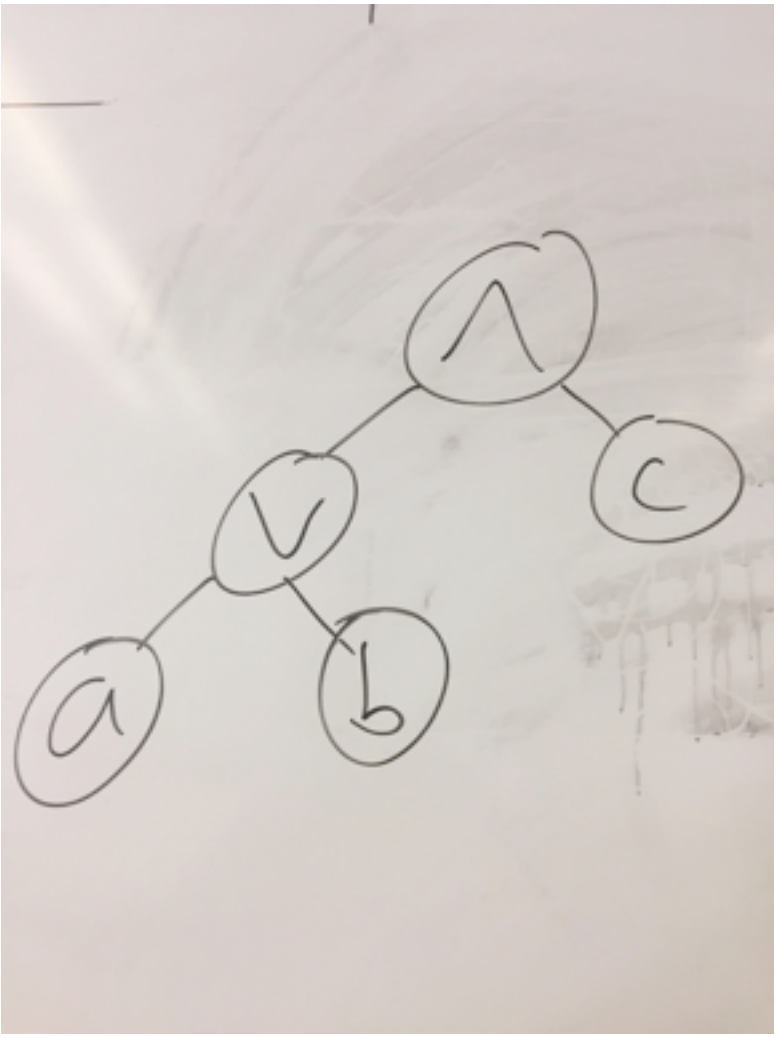
Abstract Syntax Trees

In Boolean expressions, ¬ has the highest precedence, followed by ∧ and ∨. With this in mind, write out the ASTs corresponding to each of the following Boolean expressions:

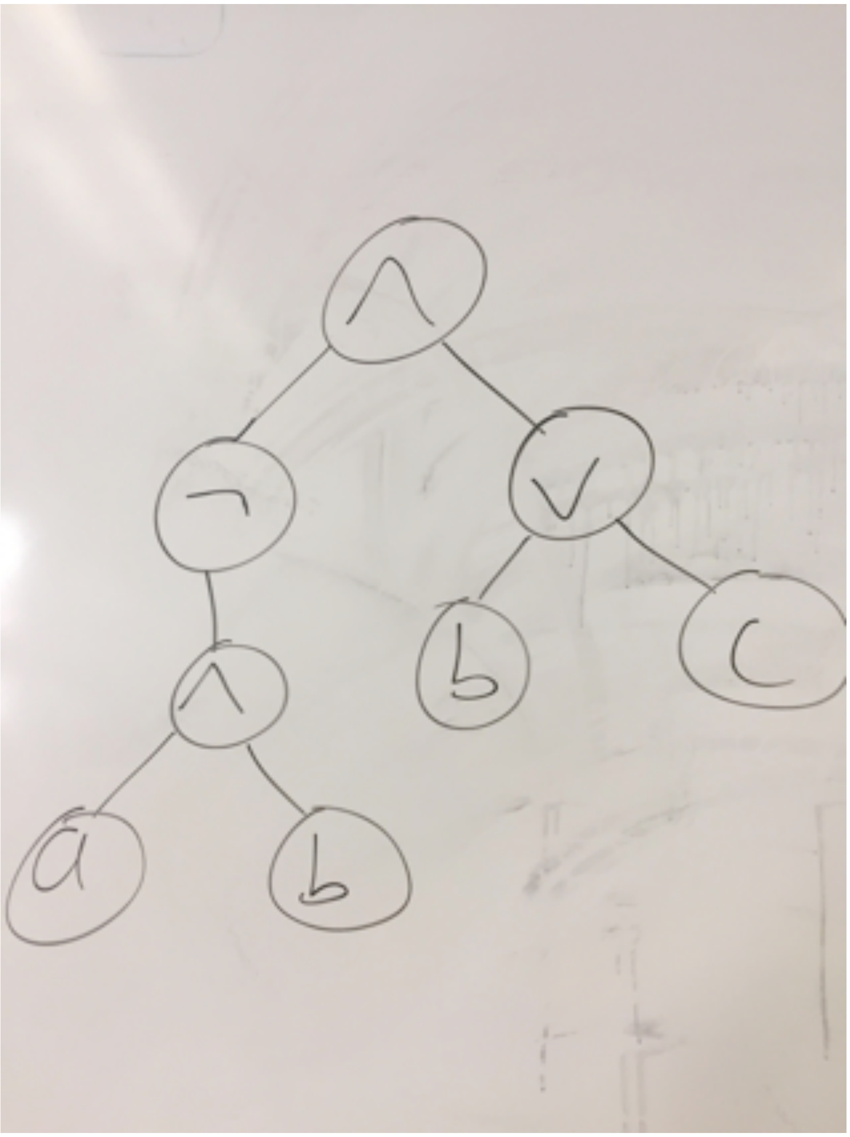
1.) `¬a ∧ b ∨ c`



2.) $(a \vee b) \wedge c$

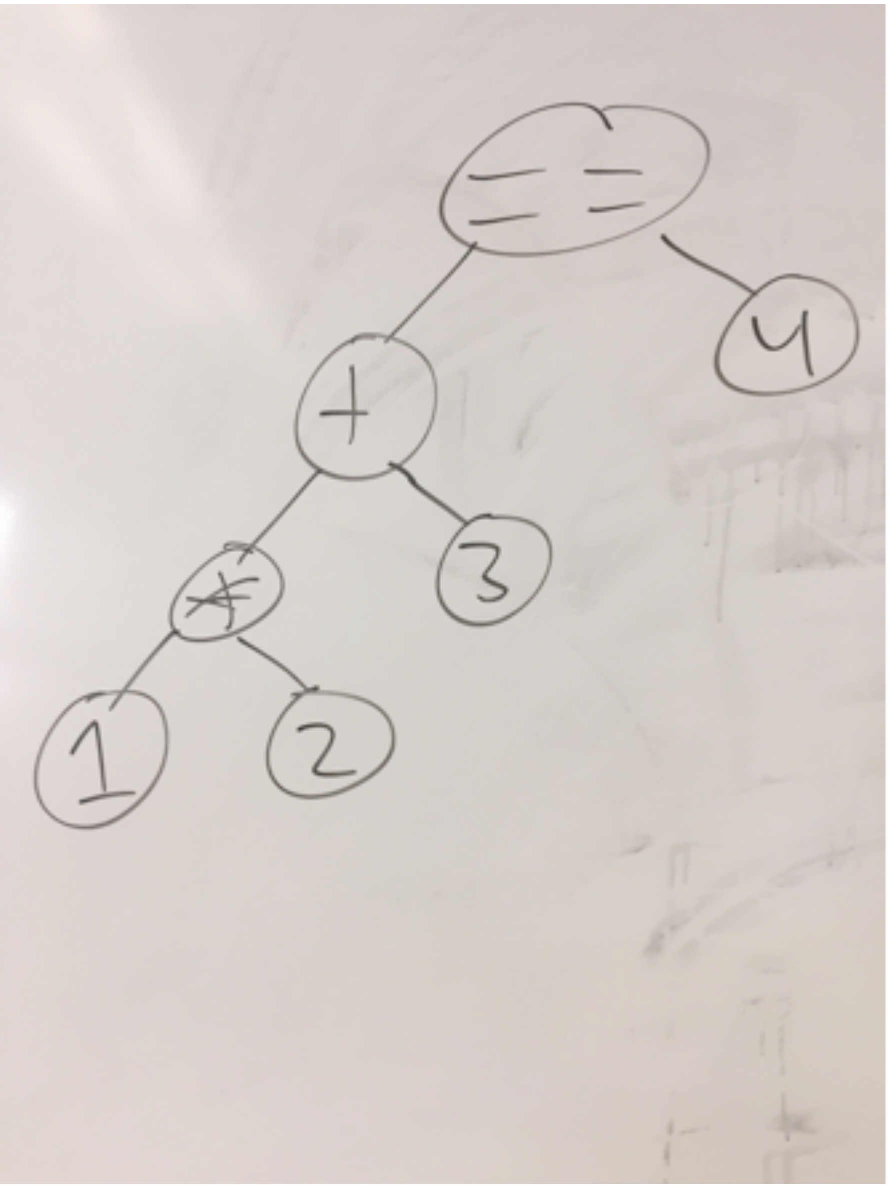


3.) ¬(a ∧ b) ∧ (b ∨ c)

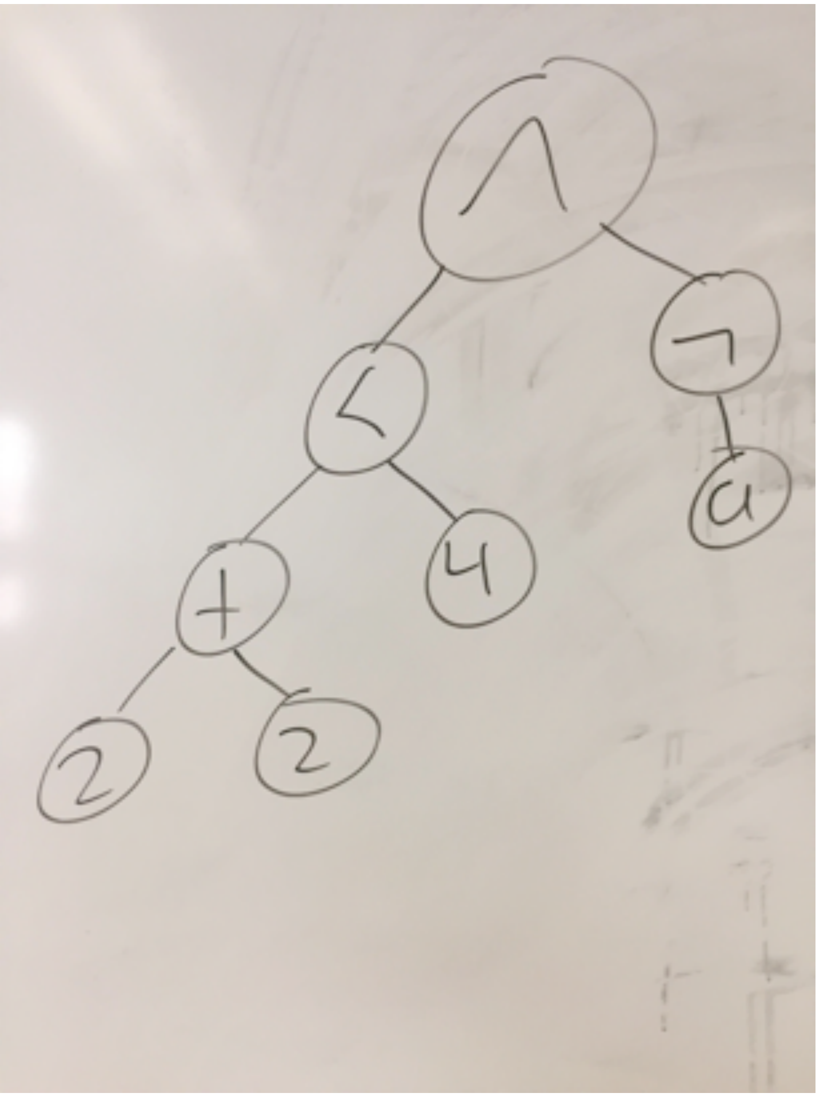


Arithmetic expressions can be used to form Boolean expressions with the help of arithmetic comparisons (e.g., <, <=, >, >=, ==). These comparisons have the lowest possible precedence. With this in mind, write out the ASTs corresponding to each of the following expressions:

4.) `1 * 2 + 3 == 4`



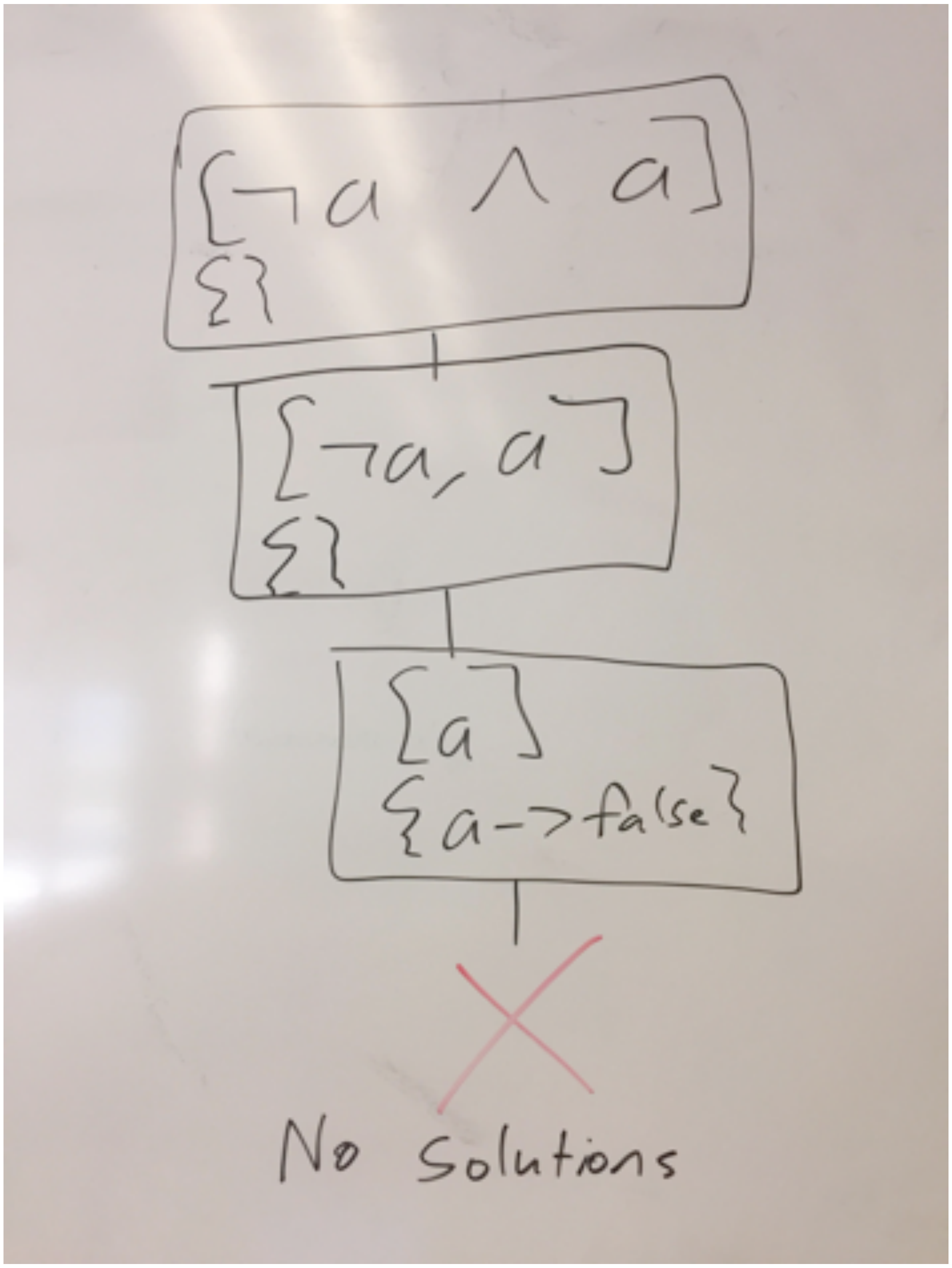
5.) (2 + 2 < 4) ∧ ¬a



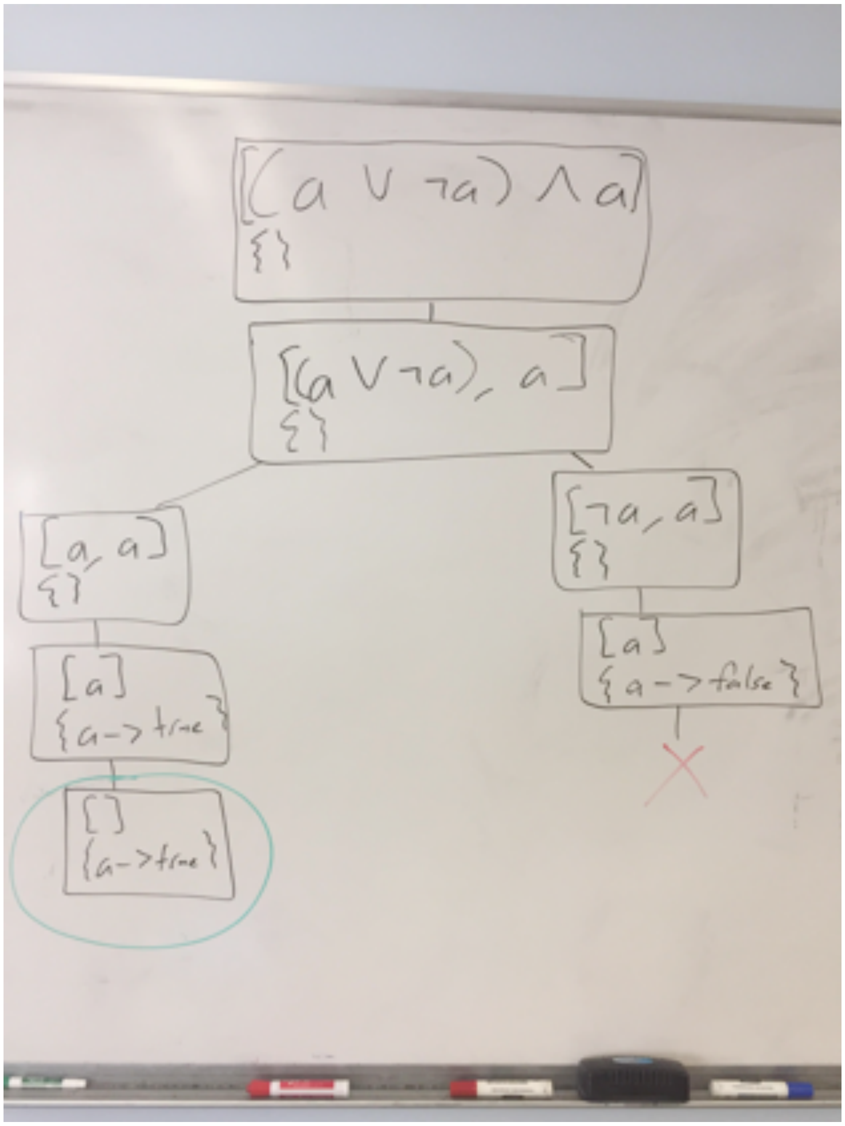
Semantic Tableau

For each of the following Boolean formulas, write out the complete semantic tableau tree. **Circle** the nodes in the tree representing solutions. If a tree has no solutions, say so. **Be sure to write all steps.**

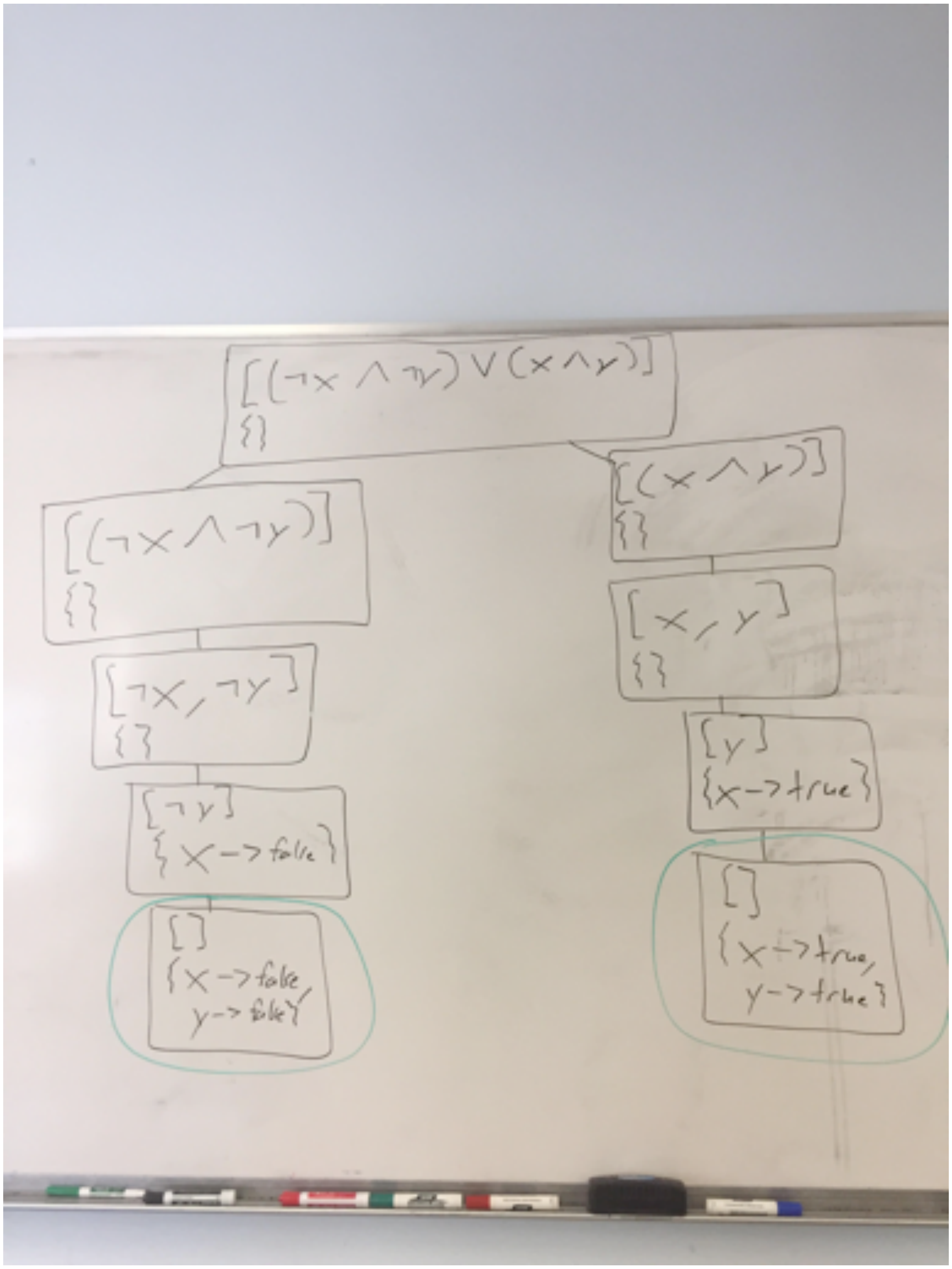
6.) ¬a ∧ a



7.) (a ∨ ¬a) ∧ a



8.) (¬x ∧ ¬y) ∨ (x ∧ y)



Prolog - Modeling the World

9.a)
For this problem, you need to write a clause database encapsulating pricing information for a convenience store. Write Prolog code accurately reflecting the following:

- Soda costs $2
- Chips cost $3
- Hot dogs cost twice as much as soda (do not hardcode $4)
- Soda chips, and hot dogs are food
- Pencils and pens are office supplies
- All office supplies cost $2
- Cold medicine costs $7

```
% all facts and rules with the same name should be placed
% together in the file
cost(soda, 2).
cost(chips, 3).
cost(hot_dog, Cost) :-
    cost(soda, SodaCost),
    Cost is SodaCost * 2.
cost(OS, 2) :-
    office_supplies(OS).
cost(cold_medicine, 7).

food(soda).
food(chips).
food(hot_dog).

office_supplies(pencil).
office_supplies(pen).
```

Using the clause database you previously wrote, write queries to determine the following:

9.b.) Which items cost exactly $2?

```
?- cost(Item, 2).
```

9.c.) Which items cost more than $3?

```
?- cost(Item, Cost), Cost > 3.
```

9.d.) Which foods cost less than $3?

```
?- food(Food), cost(Food, Cost), Cost < 3.
```

9.e.) Which foods are also office supplies?

```
?- food(Item), office_supplies(Item).
```

Prolog - Recursion

10.) Consider the following mathematical definition of a recursive function:

$$f_n = \begin{cases} 2 & \text{if } n = 0 \\ 3 & \text{if } n = 1 \\ (3 \times f_{n-1}) + (4 \times f_{n-2}) & \text{otherwise} \end{cases}$$

Write an equivalent definition in Prolog.

```
f(0, 2).
f(1, 3).
f(N, Result) :-
    N > 1,
    MinOne is N - 1,
    MinTwo is N - 2,
    f(MinOne, T1),
    f(MinTwo, T2),
    Result is (3 * T1) + (4 * T2).
```

11.) Write a procedure named `evensBetween`, which will nondeterministically produce all the even numbers within an inclusive range. As a hint, a number `N` is even if and only if the clause `0 is mod(N, 2)` is true. An example query is below:

```
?- evensBetween(1, 4, Even).
Even = 2 ;
Even = 4.
```

```
evensBetween(Min, Max, Min) :-
    Min   =< Max,
    0 is mod(Min, 2).
evensBetween(Min, Max, Result) :-
    Min   < Max,
    NewMin is Min + 1,
    evensBetween(NewMin, Max, Result).
```