CS24 Week 9 Lecture 1

Kyle Dewey

Overview

- Heaps
- Hash tables

Heaps

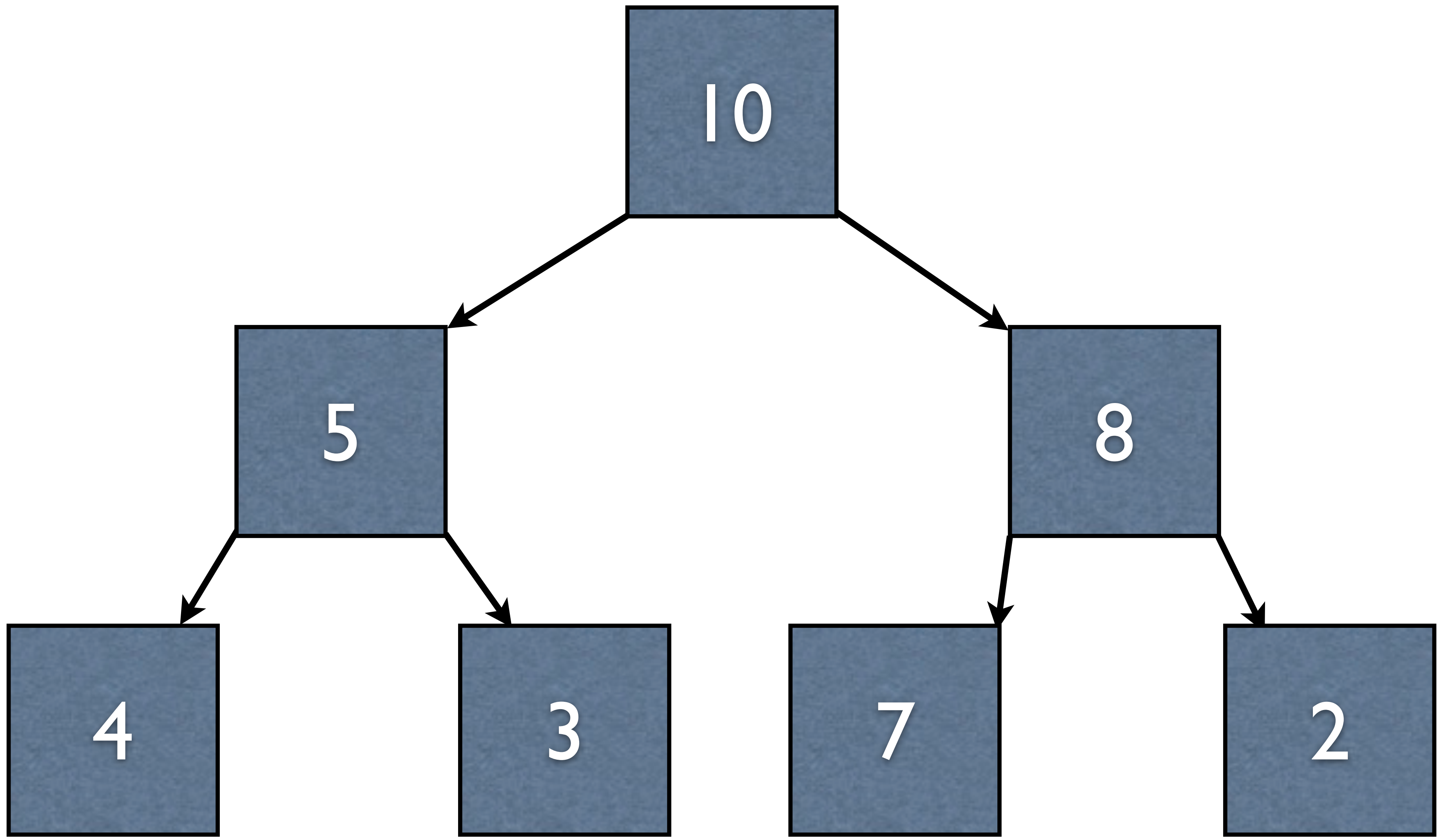
Heap

- **Not** a binary search tree; just a binary tree
- Always have the maximal (or minimal) element at the root
- Support removing the root element in `O(log(N))`, and adding elements in `O(log(N))`

Heap Property

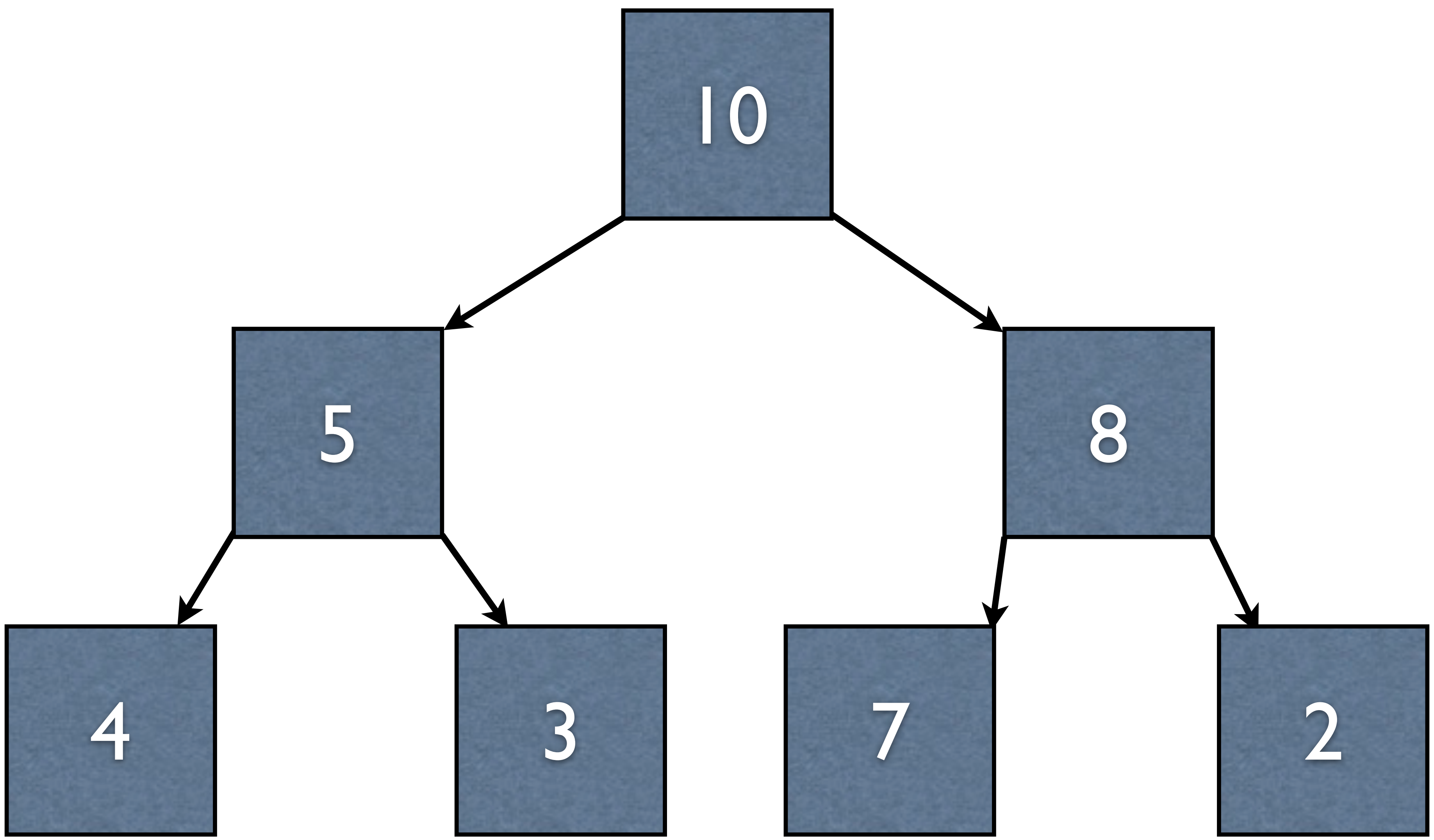
- A binary tree has the heap property if:
  - It is empty
  - Its value is greater than or equal to both of its children, and the children have the heap property

Example



Advantage

- Heaps always have the highest priority element on top, so we always have easy access to it



Additional Invariant

- In practice, heaps are always complete
  - What does this mean?

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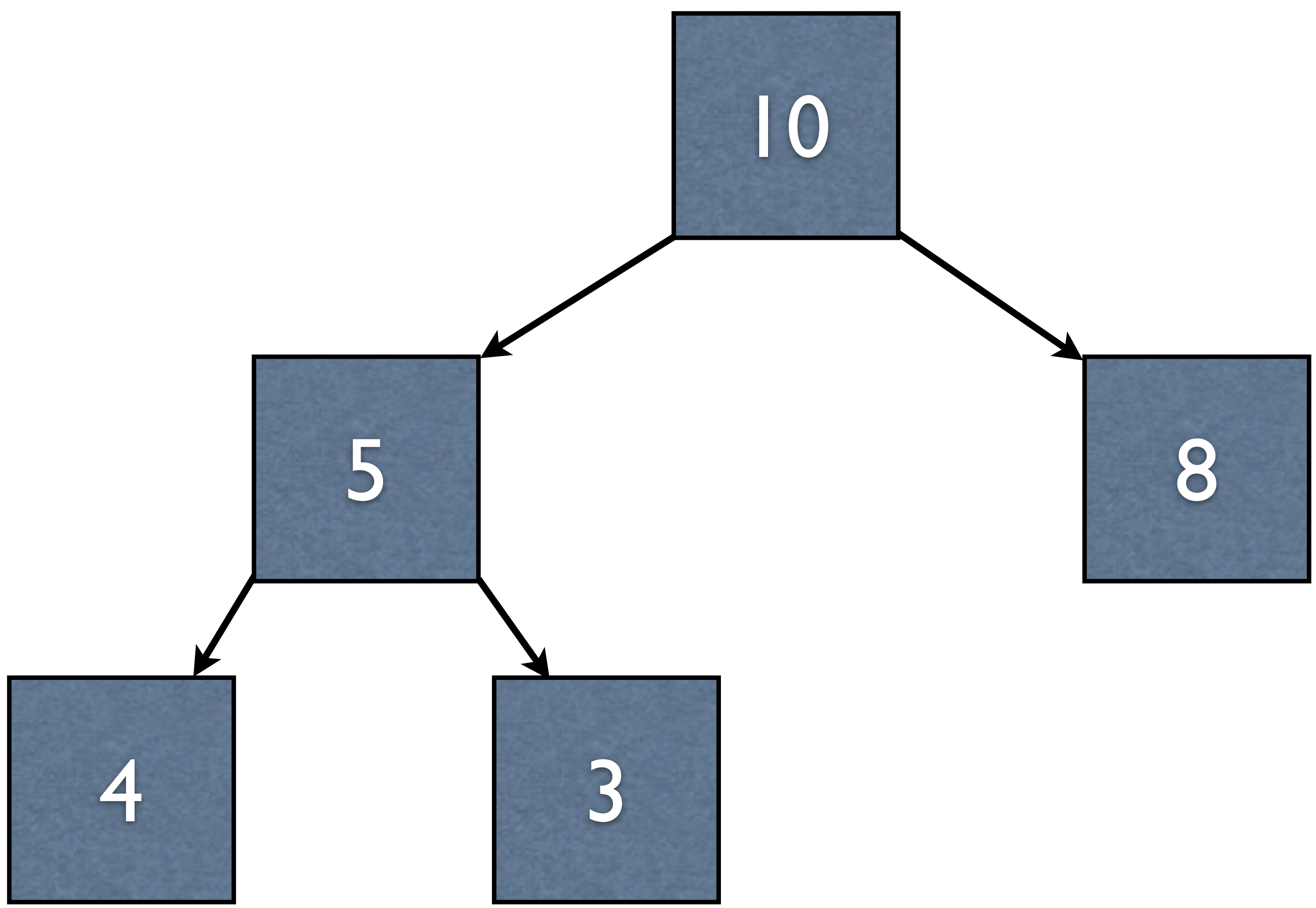
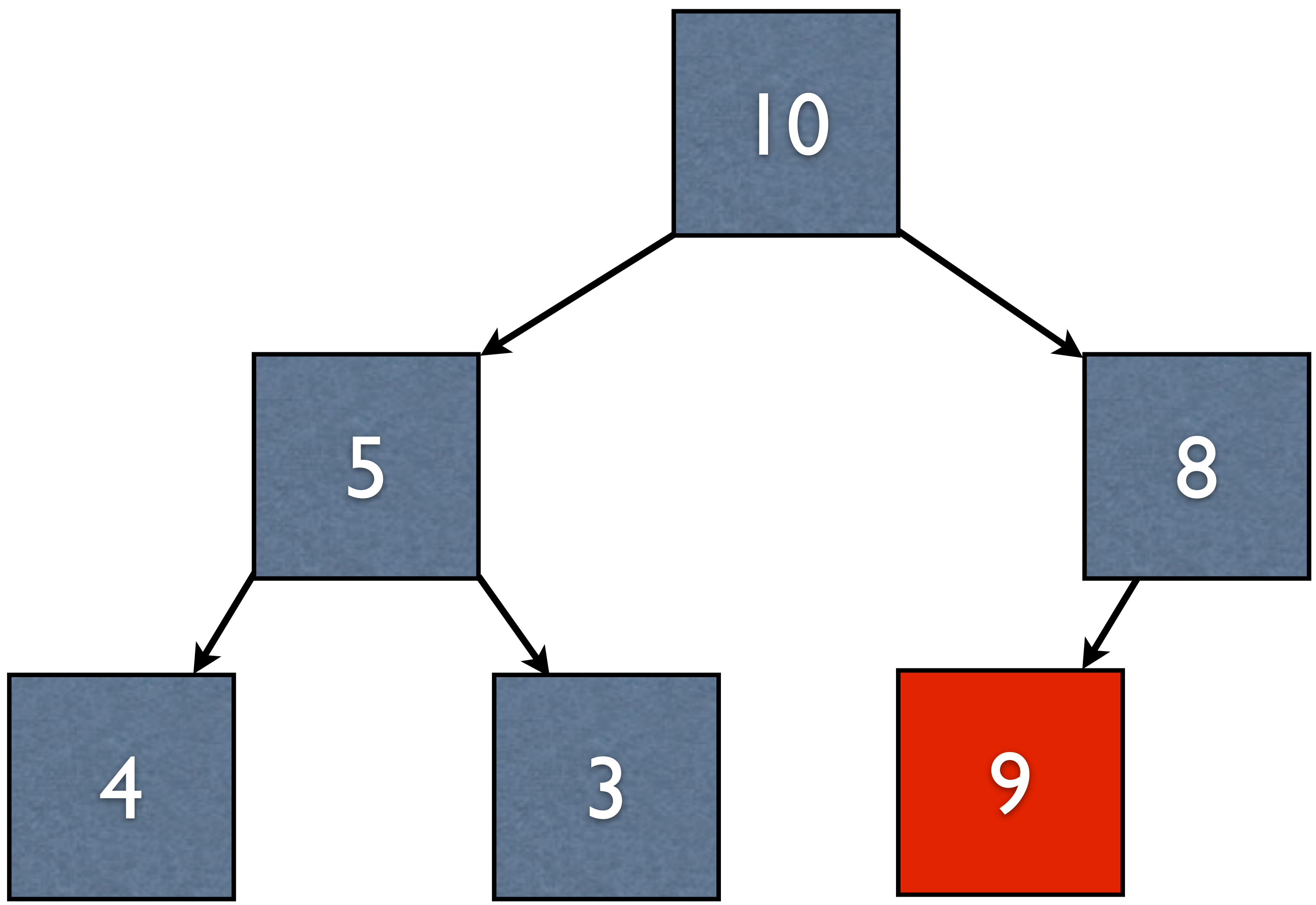
- In practice, heaps are always complete
  - What does this mean? - full except for the last row



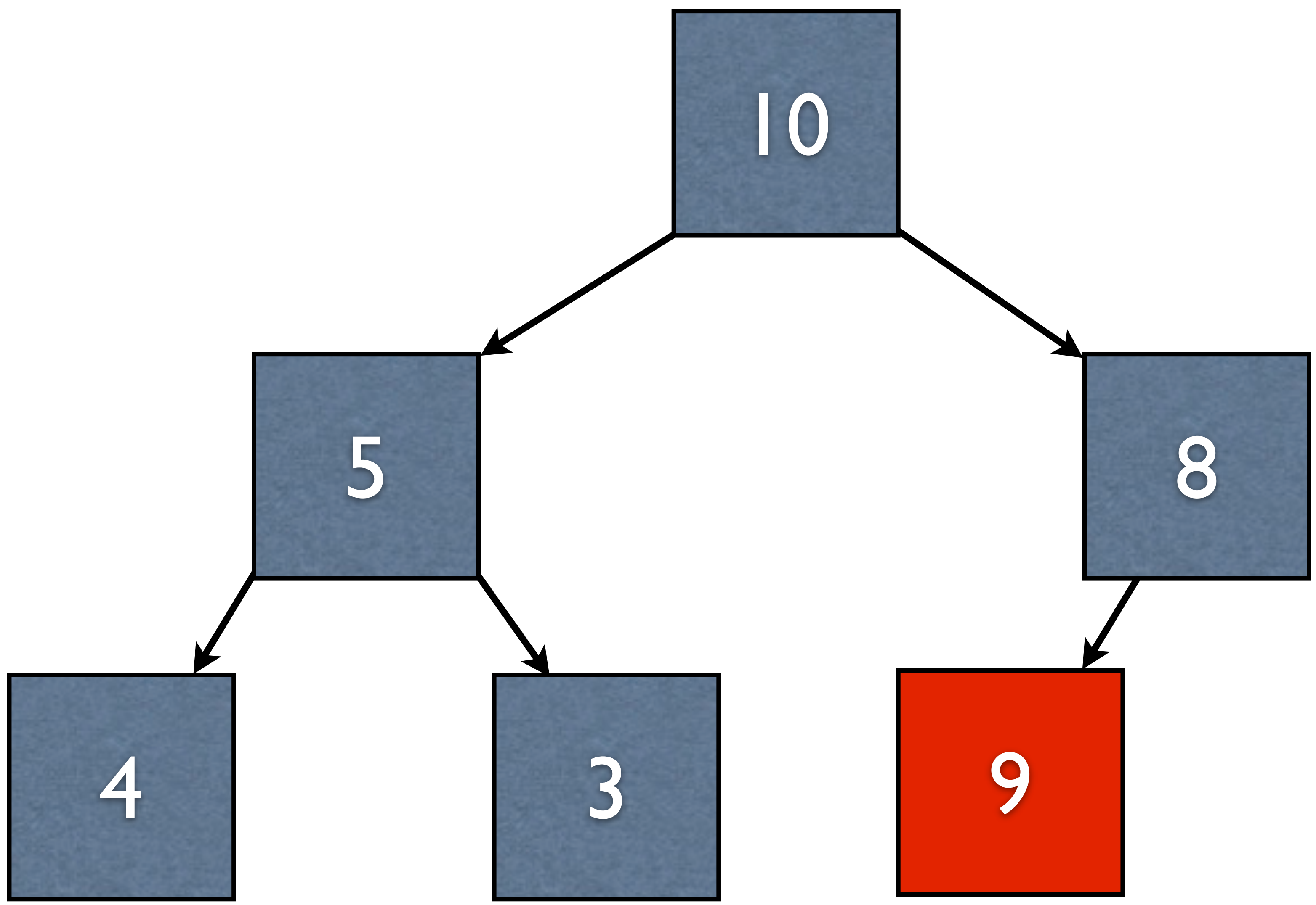
Enqueue

- If the tree is complete, we can enqueue by putting the element on the end
  - Not done yet - could violate heap property



Enqueue

- To restore the heap property, we can *bubble up* - ensure the heap property holds stepwise with parents, and swap if not



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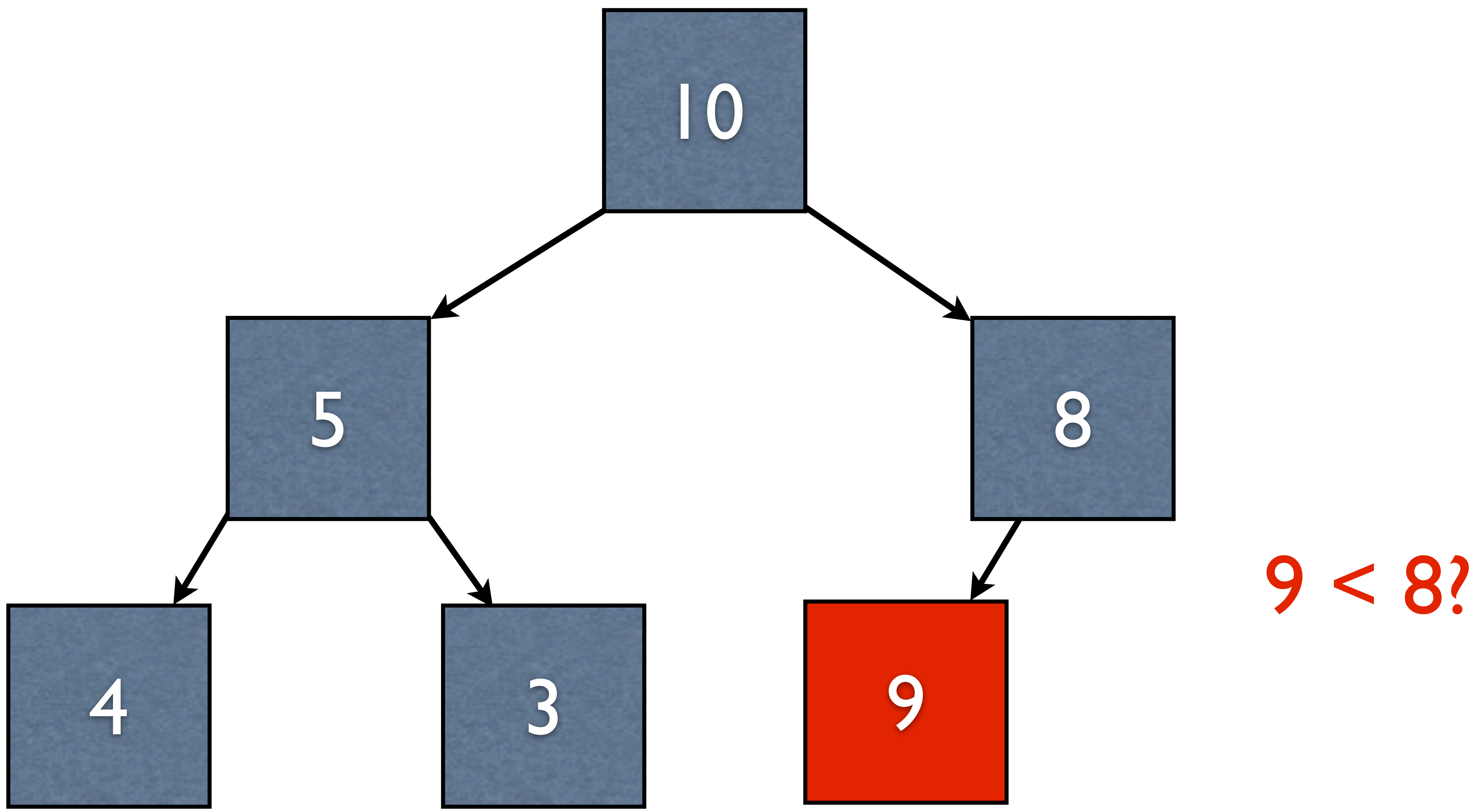

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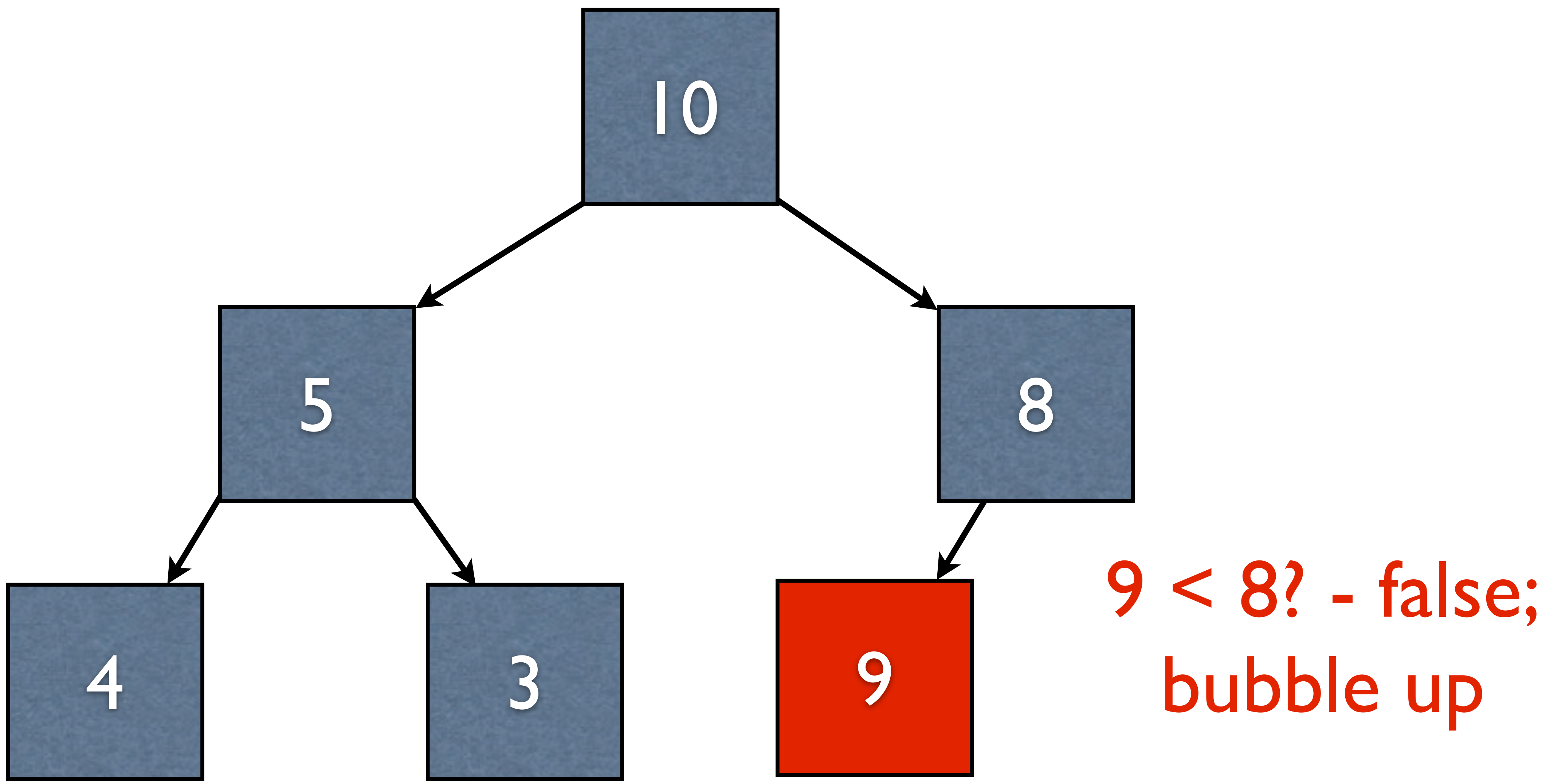

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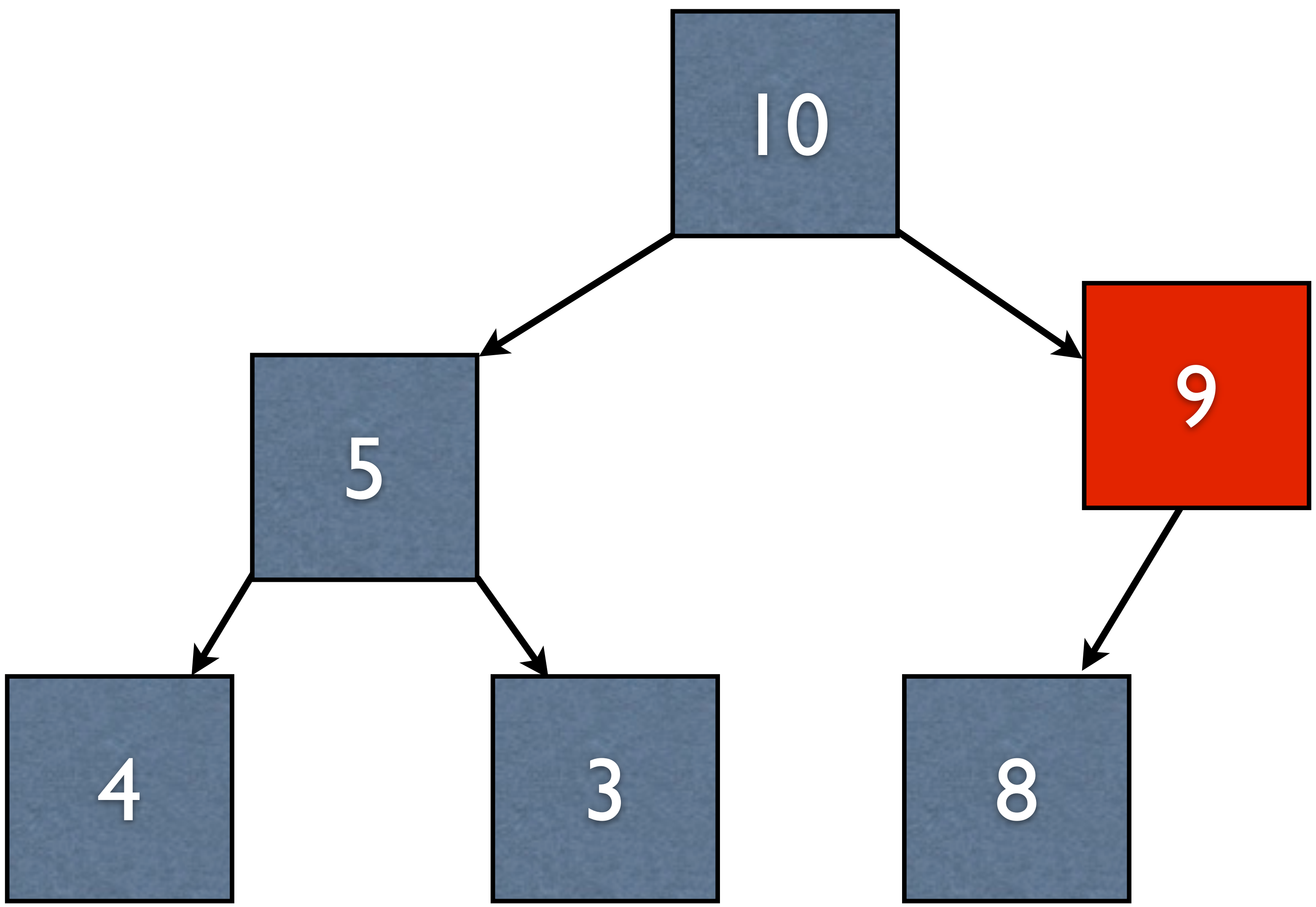

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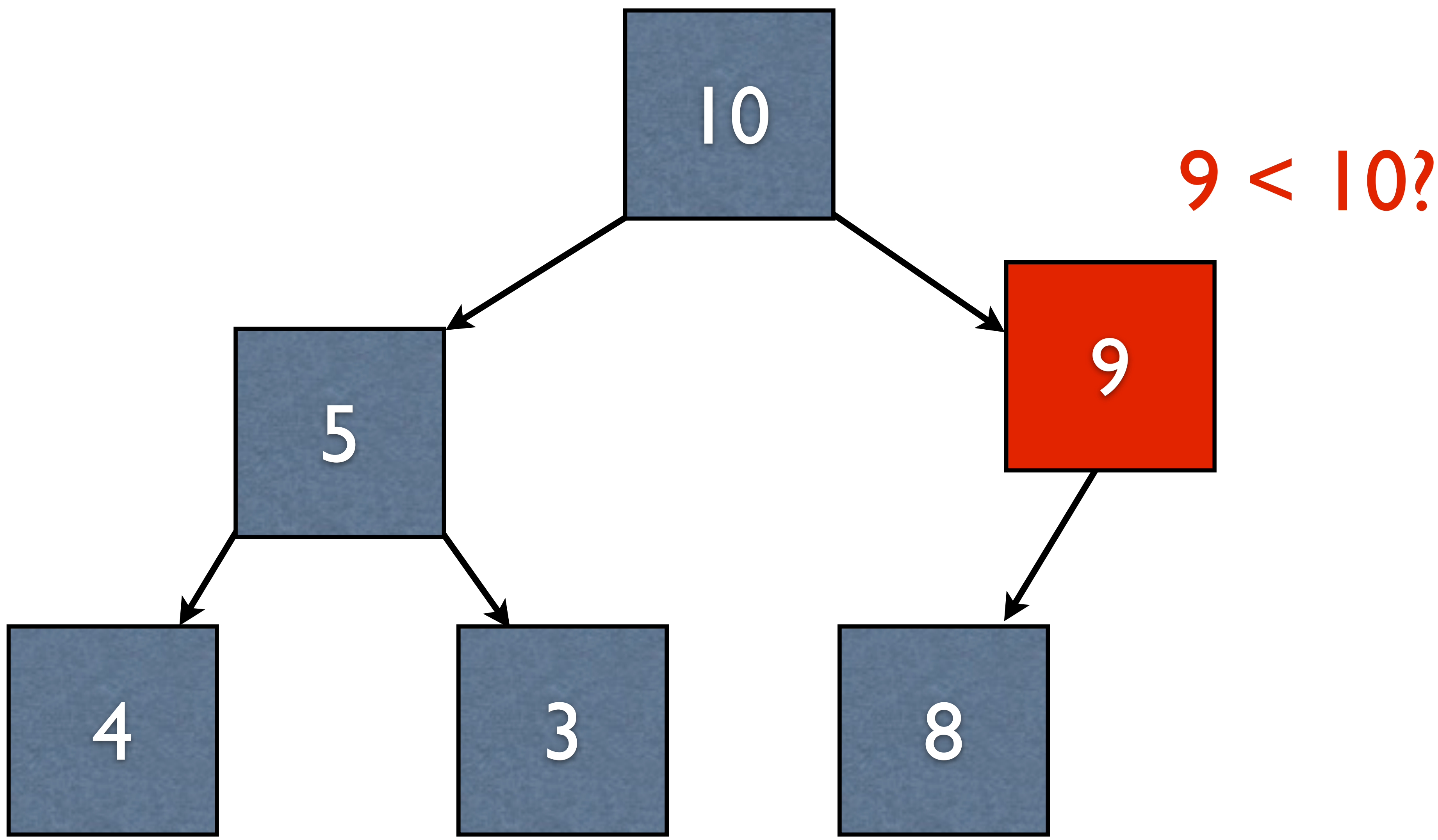

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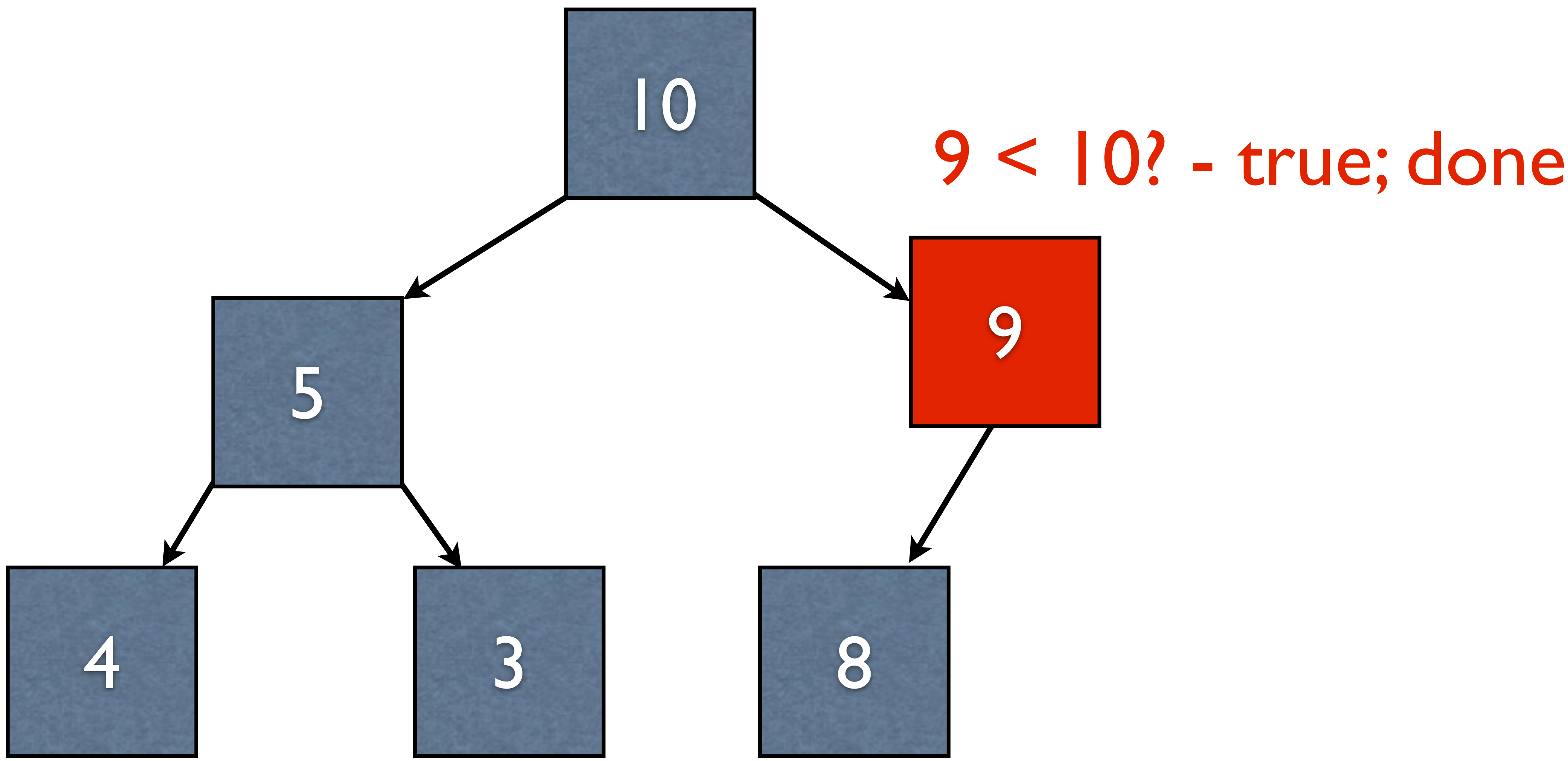
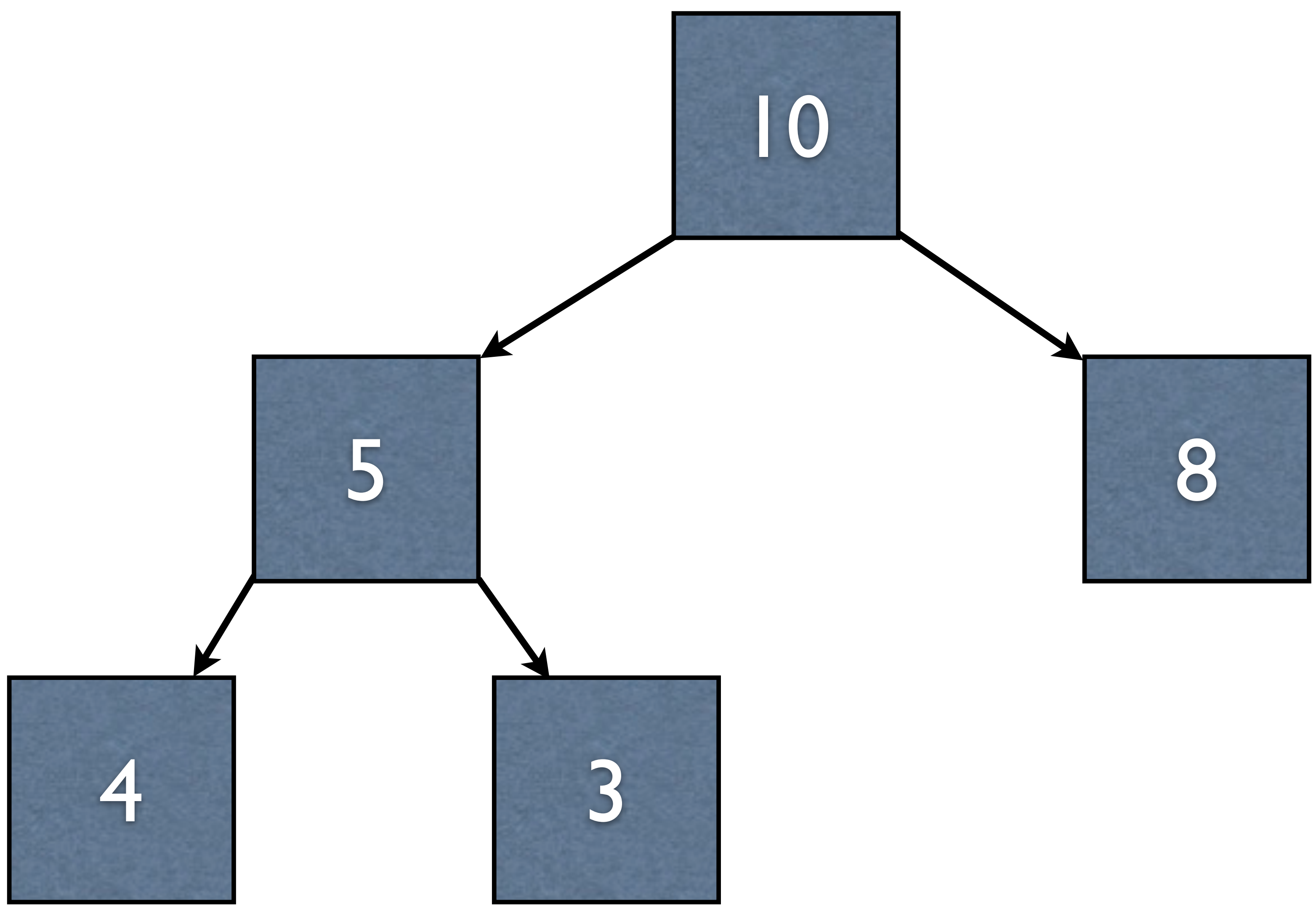


Dequeue

- After getting the element from the top of the tree, we must restore the heap



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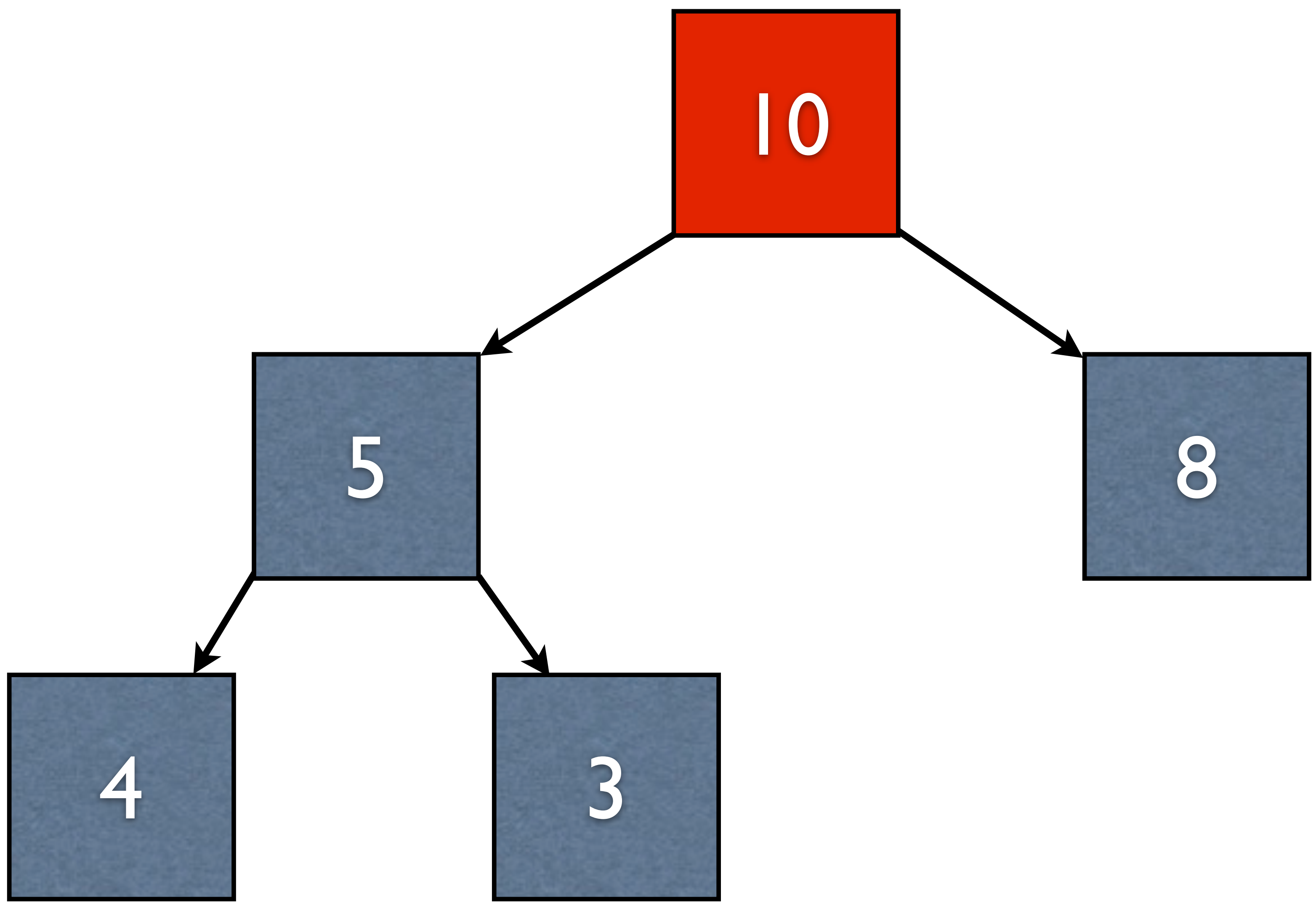

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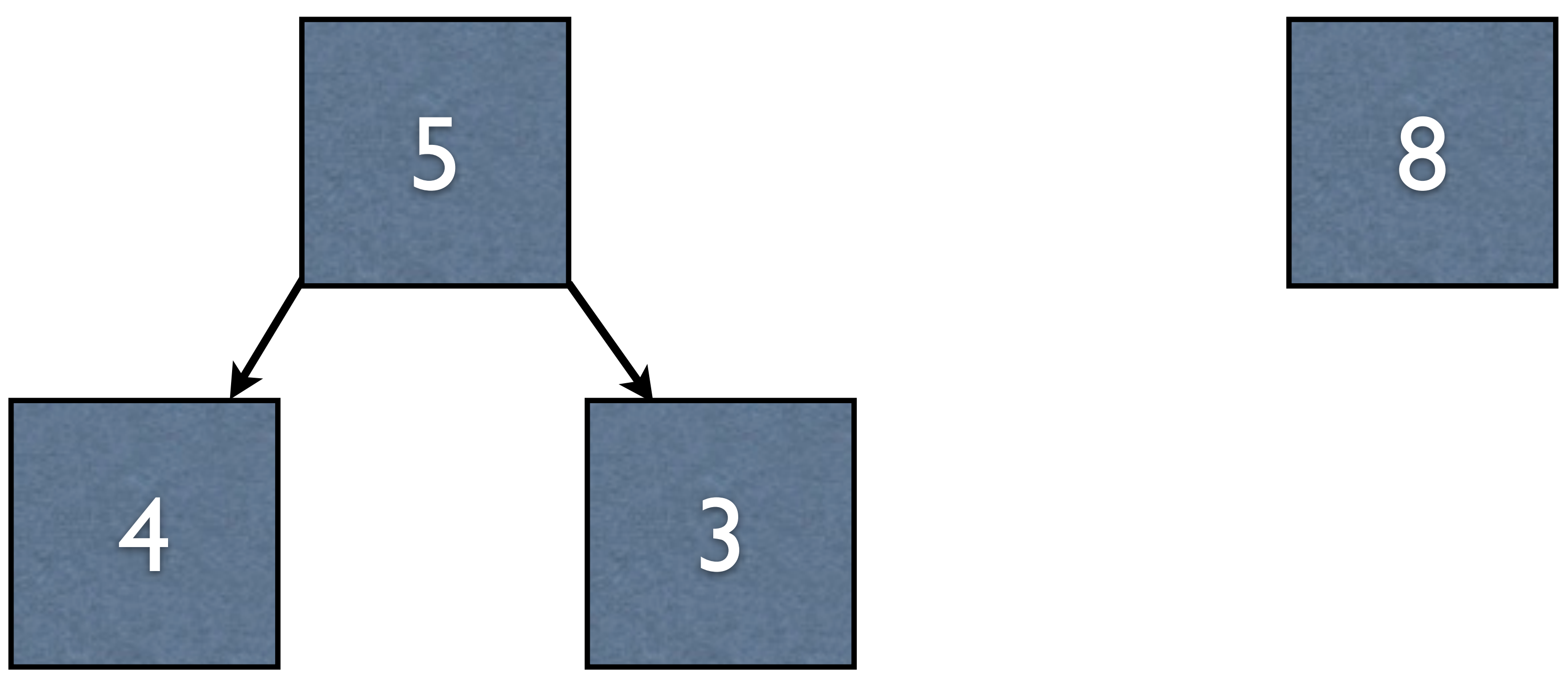

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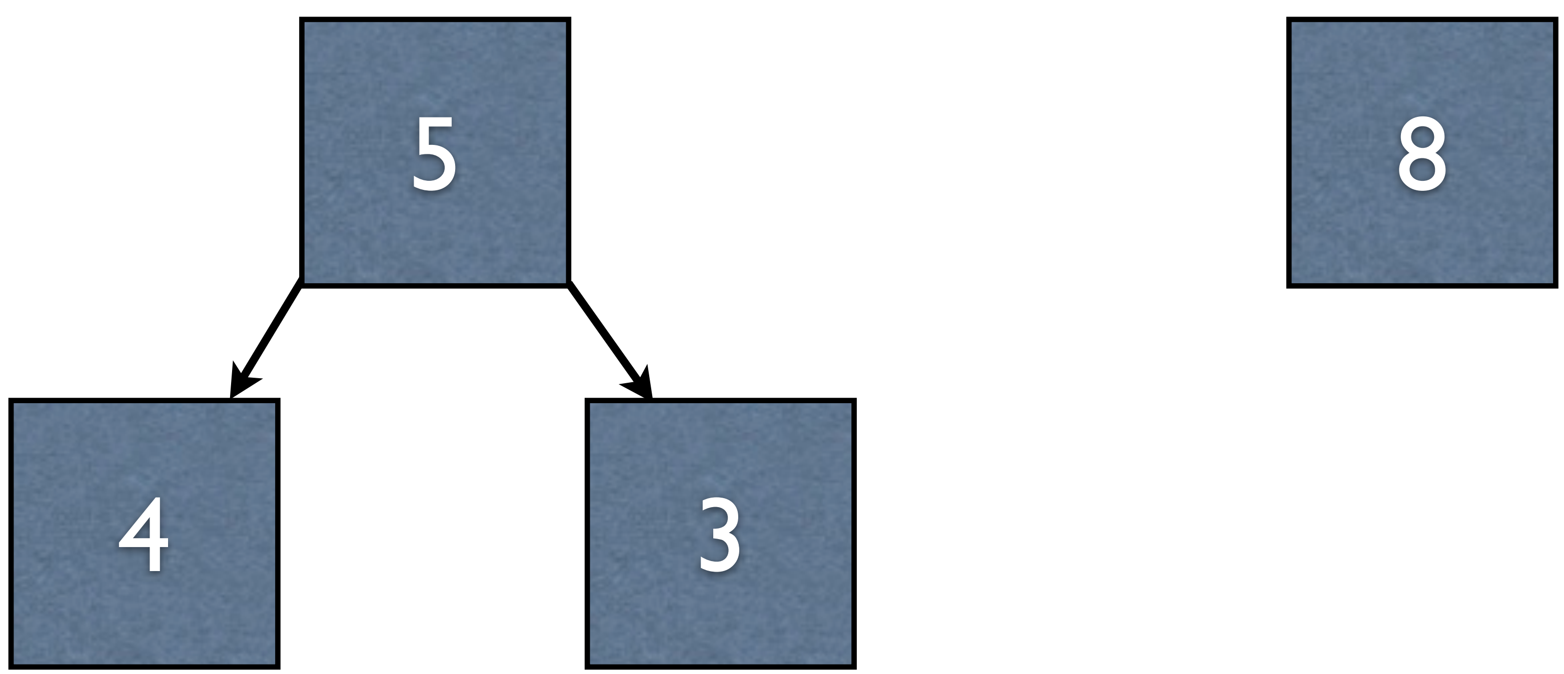
  - Idea: swap in the last node from the last level



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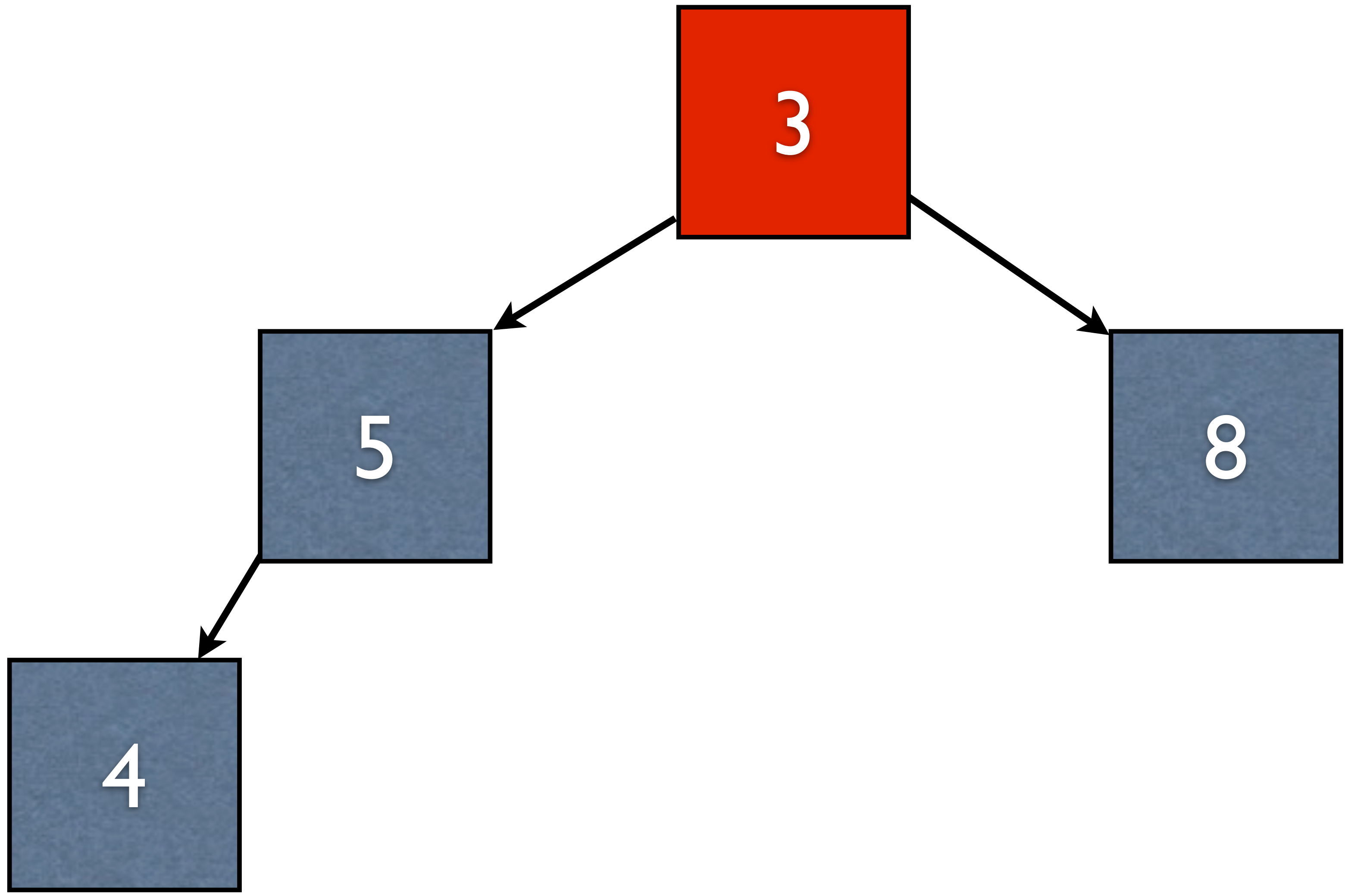
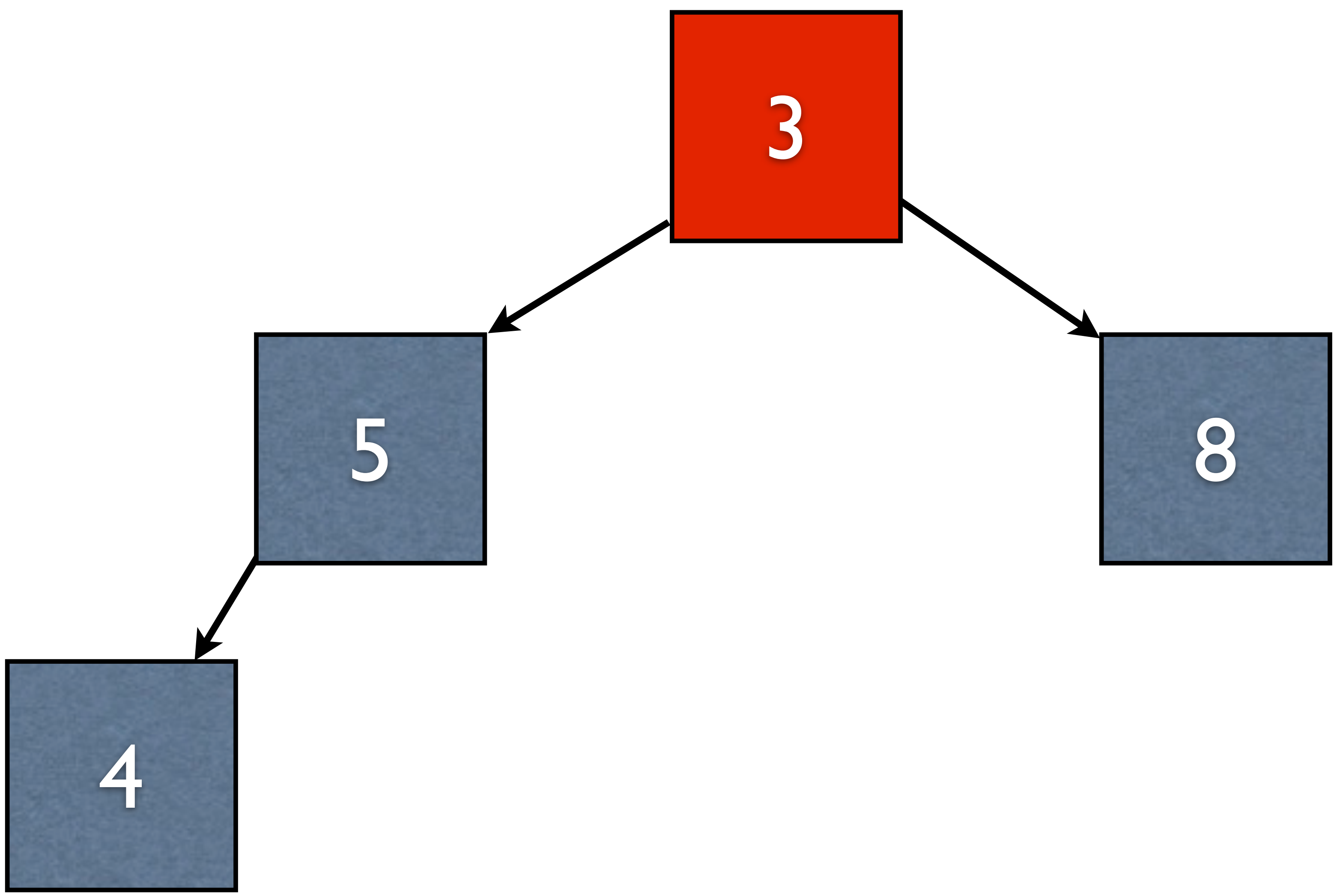


Dequeue

- In order to restore the heap property, we must *bubble down* - swap with the greatest of the children recursively



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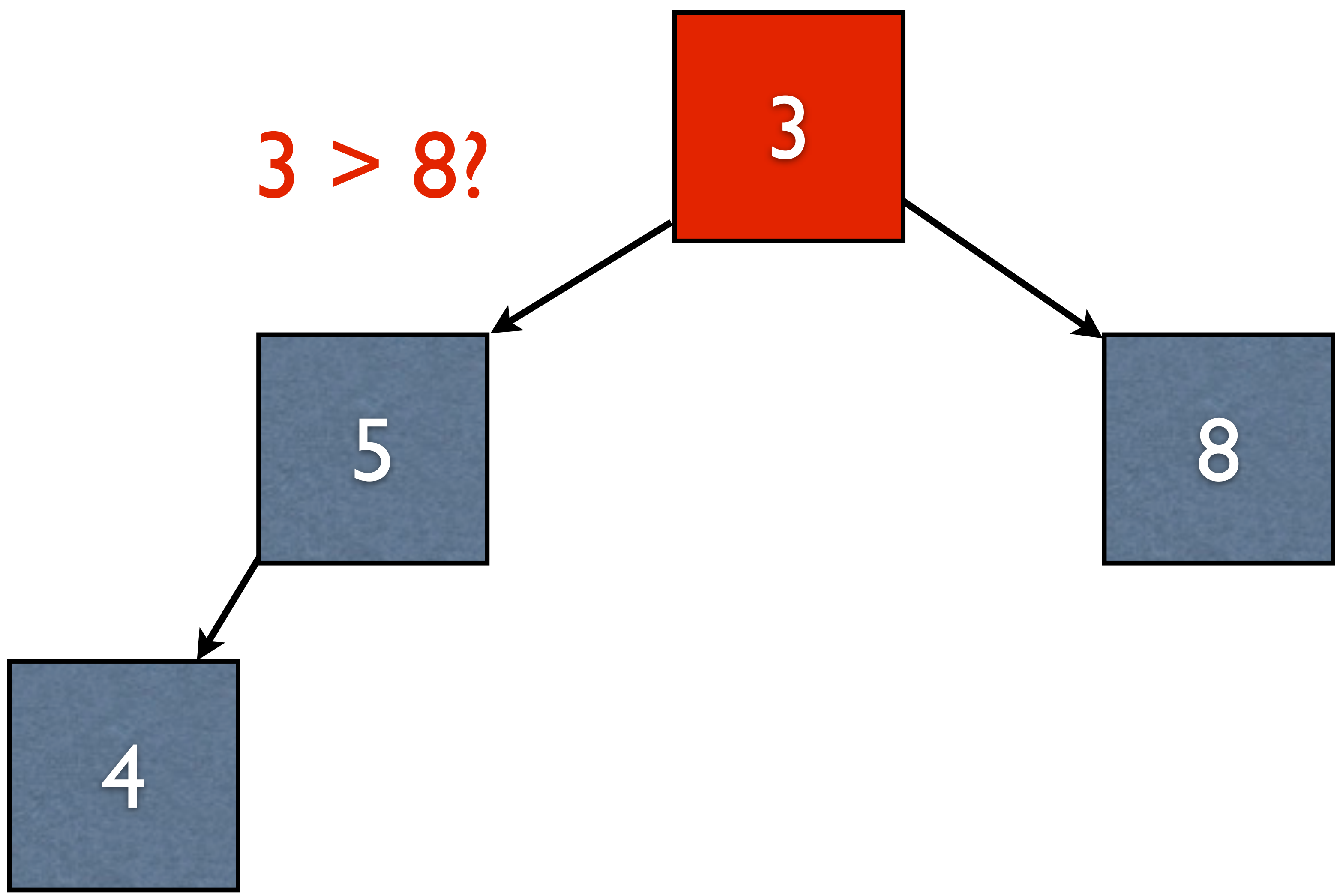

Dequeue

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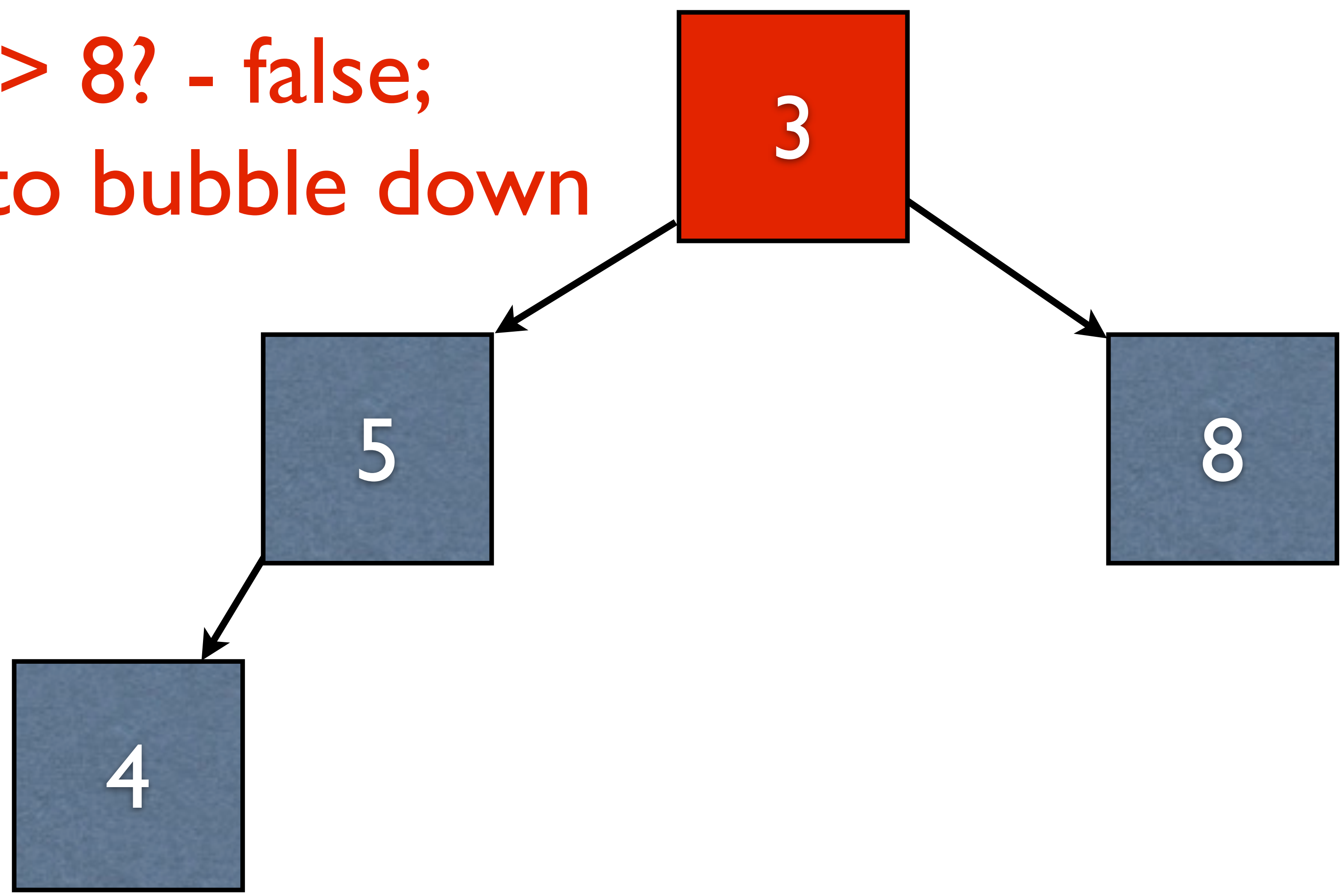
3 > 8? - false;
need to bubble down



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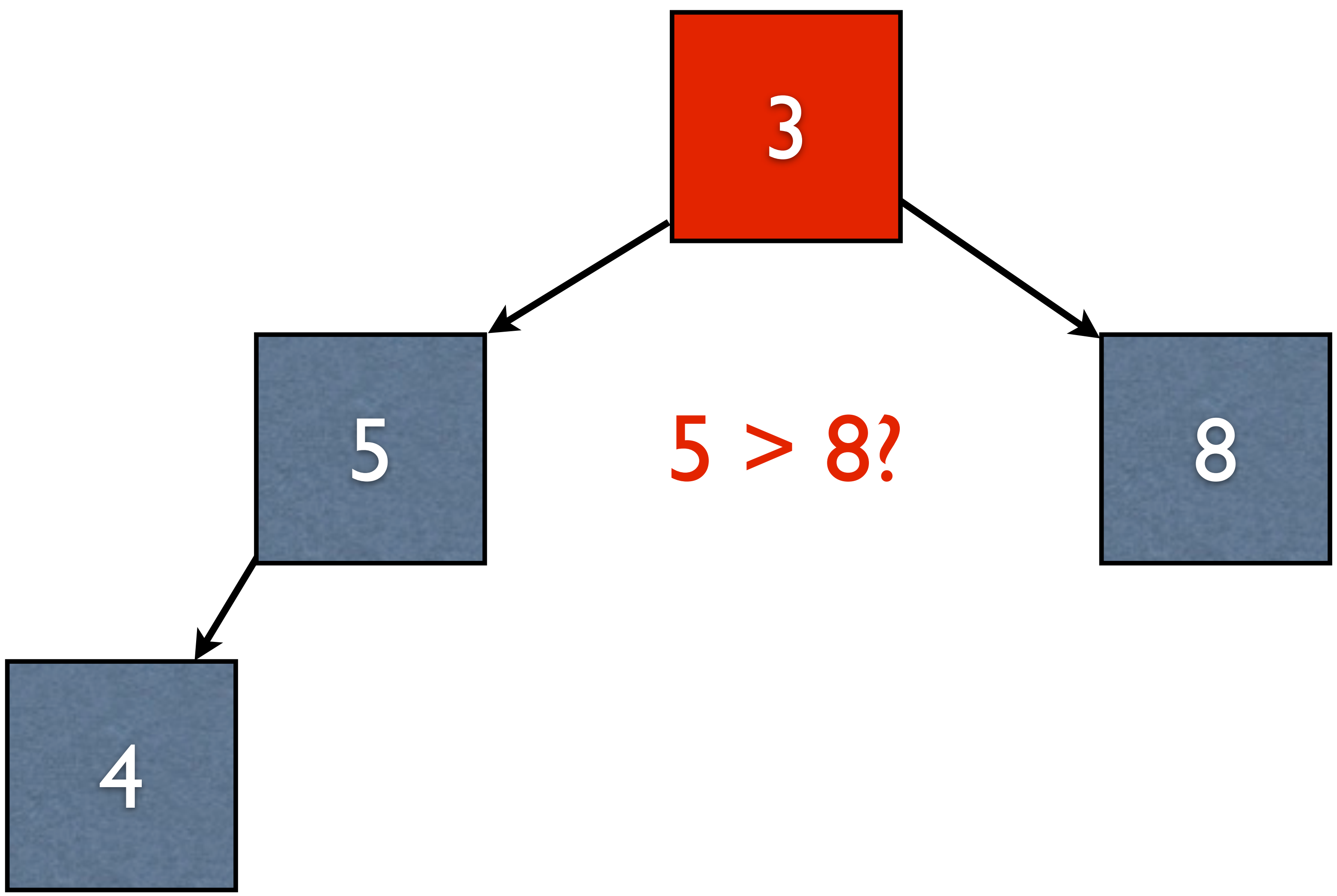

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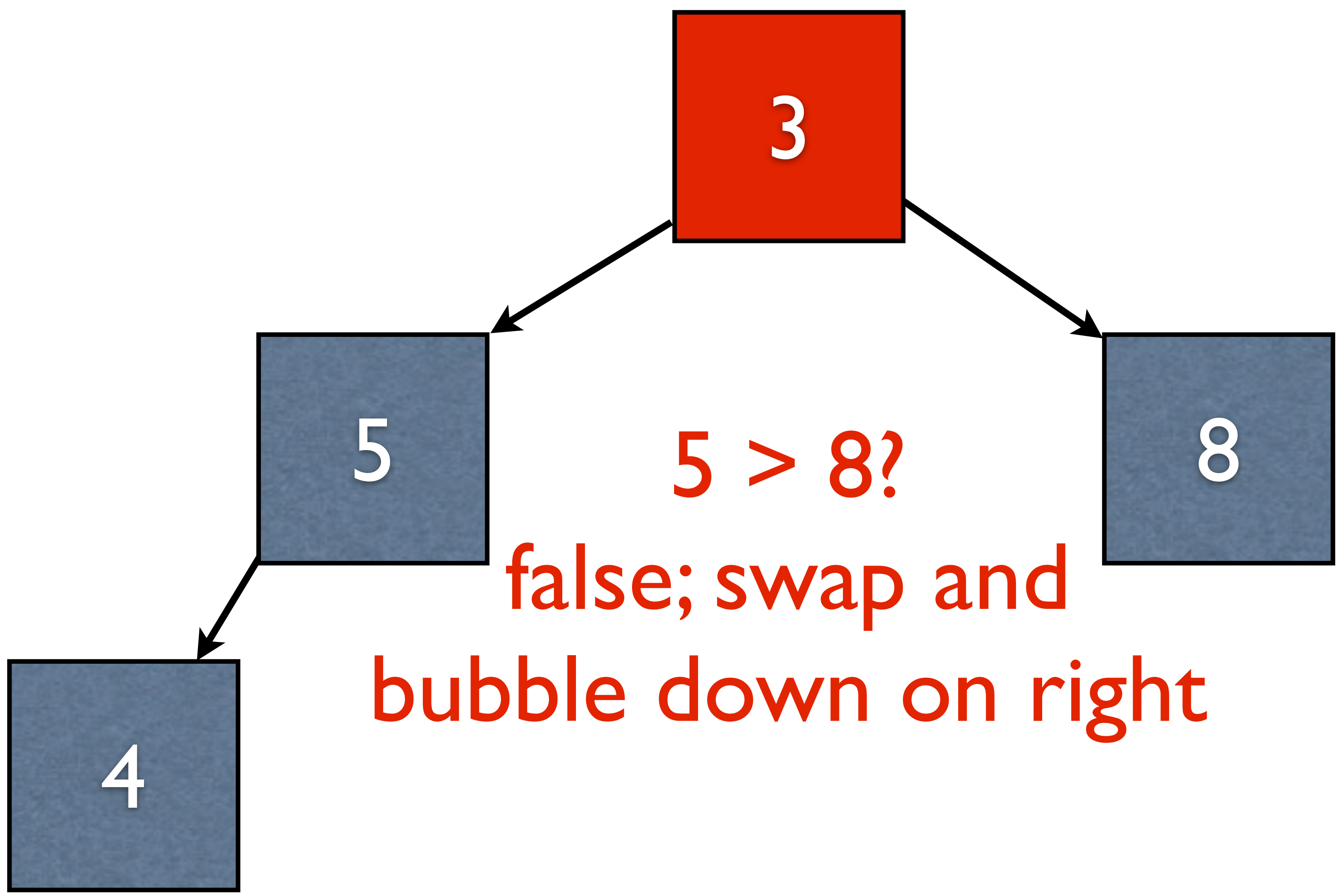

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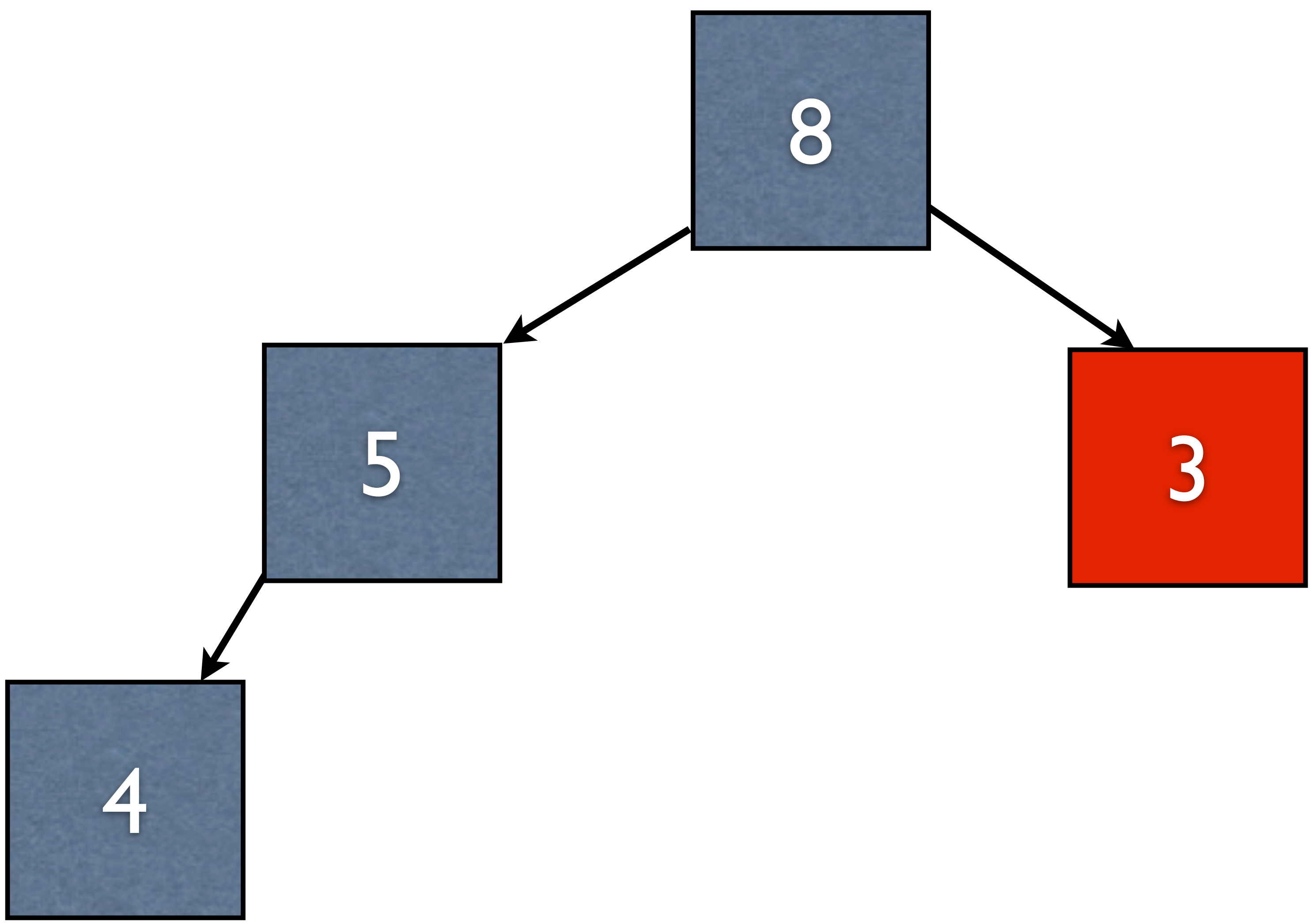

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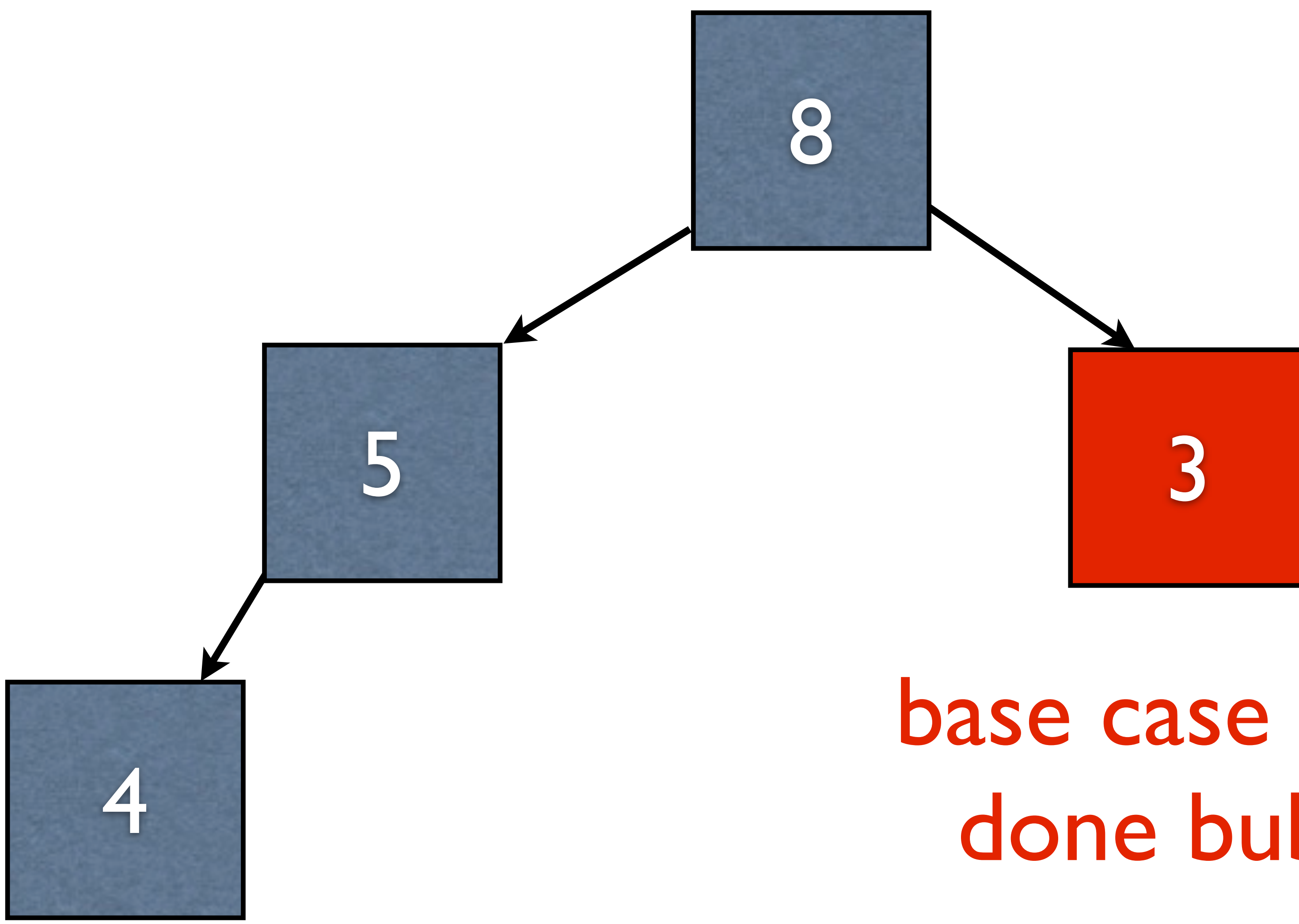


base case - no children; done bubbling down

Time Complexity

- Because we force the construction to be complete, we get balanced trees
- Dequeue and enqueue are both `O(log(N))` as a result

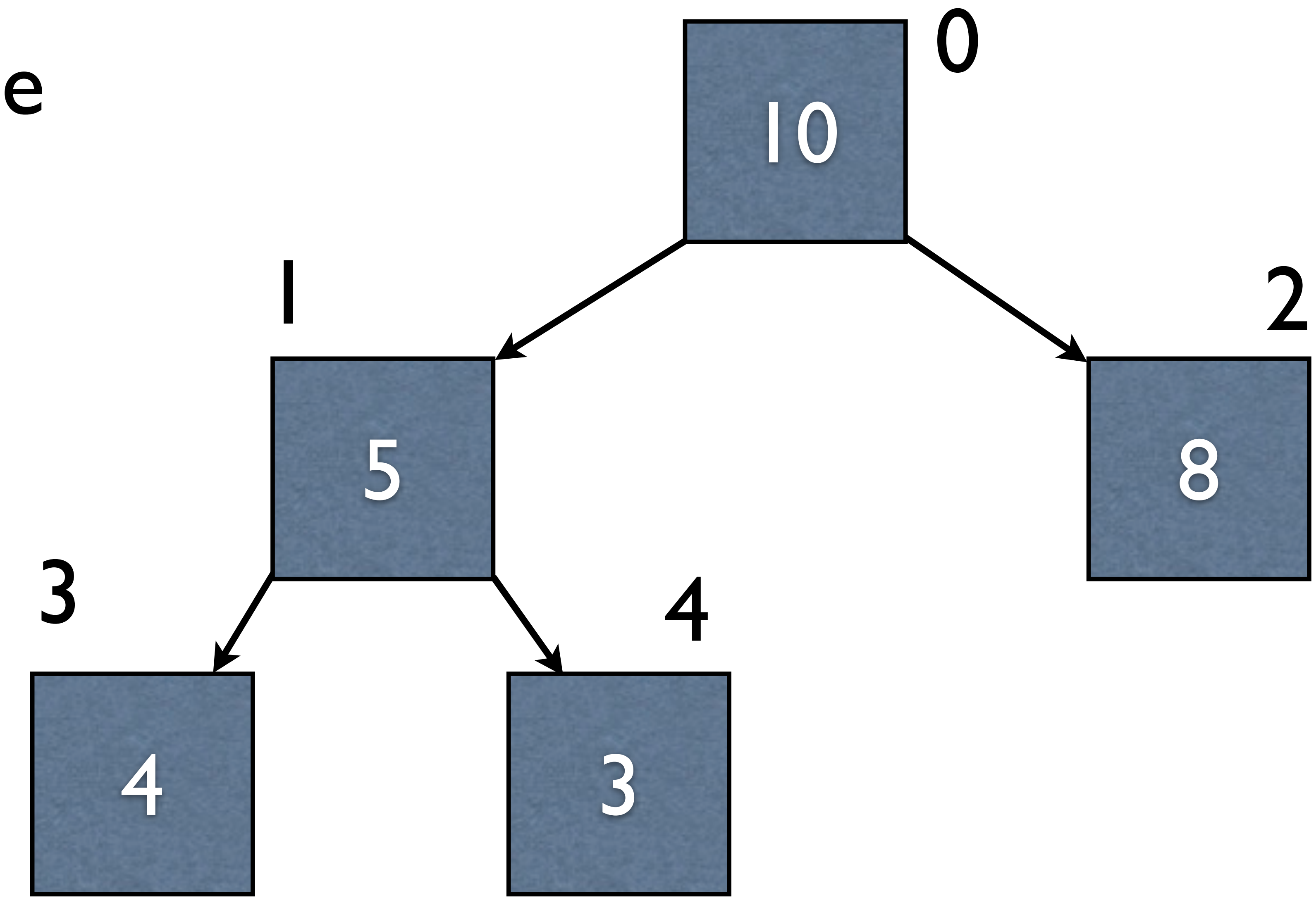
Optimization

- Heaps can be concisely represented with arrays

As Array

10	5	8	4	3

As Tree



Advantages of Arrays

- What sort of advantages does an array representation have?

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- What sort of advantages does an array representation have?
  - Overall simpler
  - Less space consumed for the same data
  - Getting the last node at the last level is just getting the last valid element in the array
  - (Advanced) CPUs are much happier with arrays than trees (i.e., better performance)

Disadvantages of Arrays

- What sort of issues does the array representation have?

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- What sort of issues does the array representation have?
  - Adding elements is more difficult; may entail reallocating the whole array
  - In practice, this is very minor compared to all the other advantages

Hash Tables

Motivation

- Maps are a very common data structure
  - Given a key, give me its corresponding value (`lookup`)
  - Add in a new value associated with some key (`add`)
  - E.g., an address book

Motivation

- We want the lookup and add operations to be as fast as possible
- How might we implement these?

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- We want the lookup and add operations to be as fast as possible
- How might we implement these?
  - Could use a binary search tree - `O(N)`
  - Force the tree to be balanced - `O(log(N))`

Tree Style

- We could get `O(log(N))` performance
- Still some issues - what?

Tree Style

- We could get `O(log(N))` performance
- Still some issues - what?
  - Need to perform `O(log(N))` comparisons, and comparisons may not be cheap
  - Performance-wise, `O(1)` would be better

Doing Better

- What data structure is needed for `O(1)` lookups?

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- What data structure is needed for `O(1)` lookups?
  - Arrays

Using Arrays

- Not obvious how we might utilize arrays for this
- First, a simplifying assumption: all keys are integers `>= 0`
  - How can we take advantage of this?

Using Arrays

- Not obvious how we might utilize arrays for this
- First, a simplifying assumption: all keys are integers `>= 0`
  - How can we take advantage of this?
    - Use keys as indices!

Example

- The following example uses integers >=0 for keys and characters for values

Example

Initial array contents: all -1 (indicator that the space is unused)

-1	-1	-1	-1	-1	Array
0	1	2	3	4	Indices

Example

```
insert(3, ‘g’)
```

-1	-1	-1	-1	-1	Array
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Example

`insert(3, ‘g’)`

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Example

```
insert(1, ‘f’)
```

-1	-1	-1	g	-1	Array
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Example

`insert(1, ‘f’)`

-1	f	-1	g	-1	Array
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Example

`insert(10, 'k')`

Array	-1	f	-1	g	-1
Indices	0	1	2	3	4

Example

No index 10!
What do we do?

```
insert(10, 'k')
```

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Fixing Index Out of Bounds

- We might have a key whose index is out of bounds for the array
- How can we fix this?

Fixing Index Out of Bounds

- We might have a key whose index is out of bounds for the array
- How can we fix this?
  - Resizing is suboptimal - may have key 100,000
  - Modular arithmetic - insert at `key % arraySize`, which guarantees it will be in bounds

Example

```
insert(10, 'k')
```

No index 10! What do we do?

-1	f	-1	g	-1	Array
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Example

```
insert(10, ‘k’)

10 % 5 == 0
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Array	-1	f	-1	g	-1
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Example

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insert(10, 'k')

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k	f	-1	g	-1	Array
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Example

`insert(11, ‘o’)`

Array	k	f	-1	g	-1
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Example

`insert(11, ‘o’)`

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Example

Problem - we already have something at 1. Additionally, f was inserted with a different key (1). Both now belong at this position.

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insert(11, 'o')

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Collision Problem

- We have multiple entries that belong in the same slot, even though they have different keys
  - Downside of using modular arithmetic
- How might we fix this?

Collision Problem

- We have multiple entries that belong in the same slot, even though they have different keys
  - Downside of using modular arithmetic
- How might we fix this?
  - Store a linked list at this position of key/value pairs

Example

Problem - we already have something at 1. Additionally, f was inserted with a different key (1). Both now belong at this position.

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insert(11, 'o')

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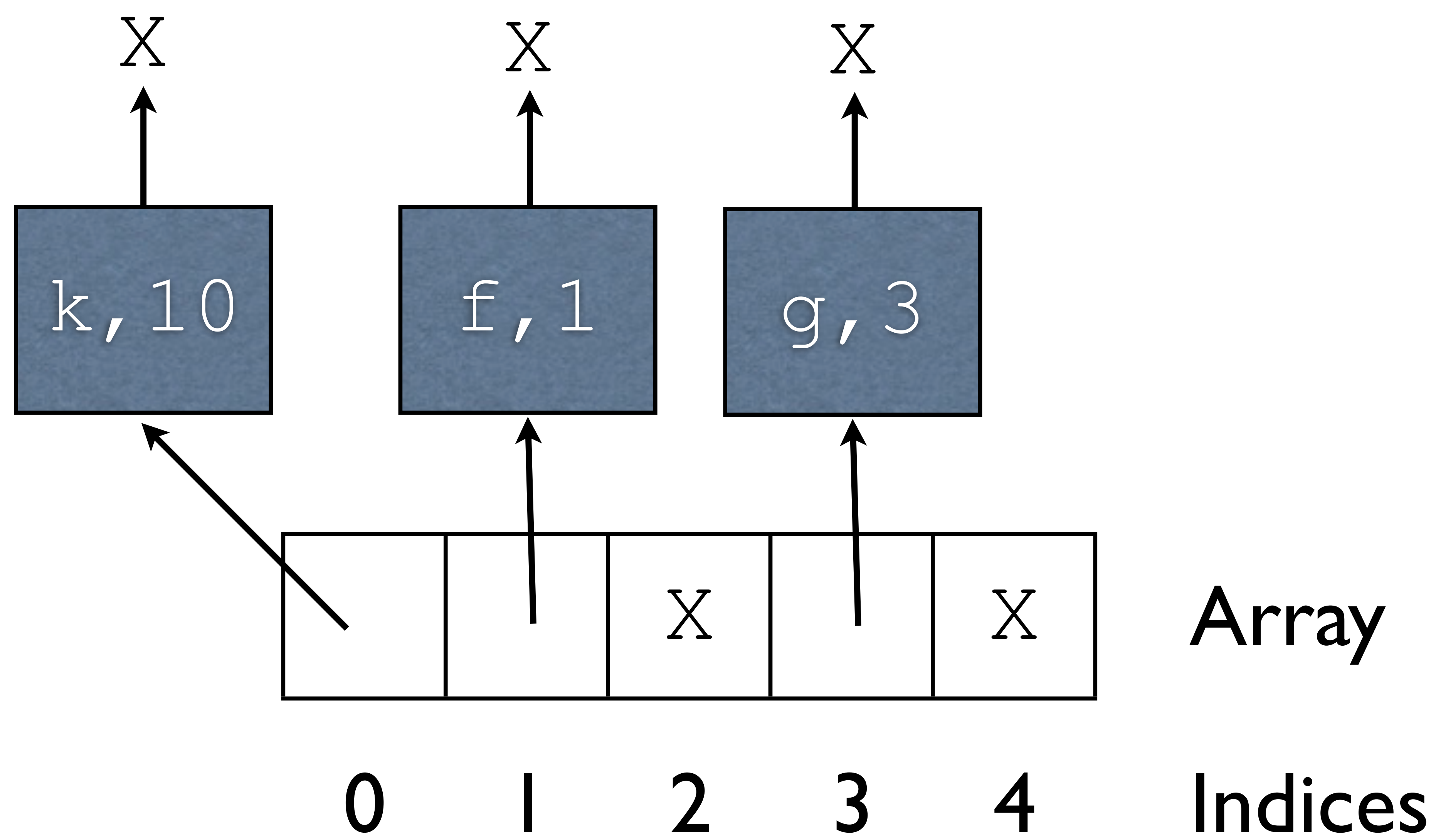

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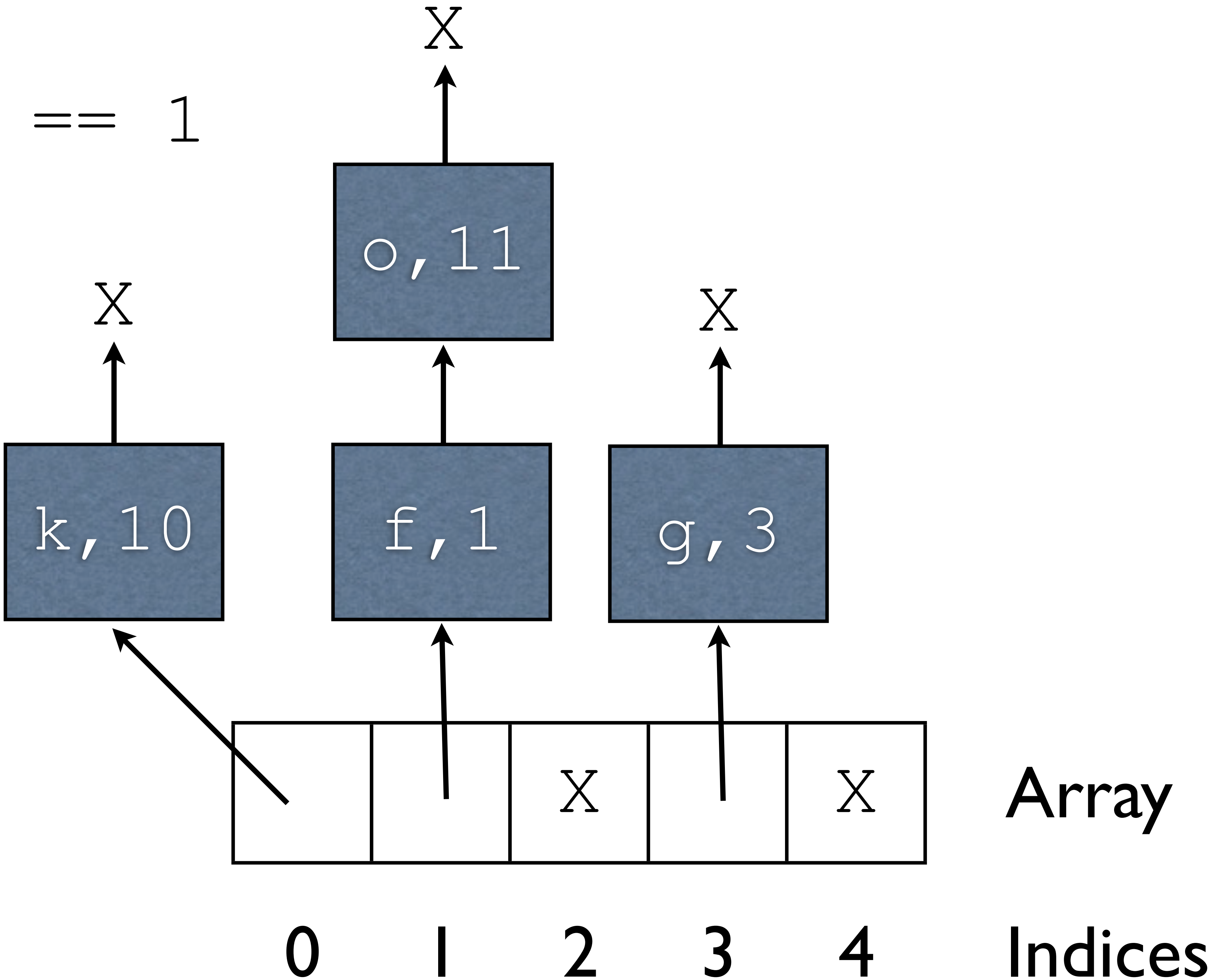
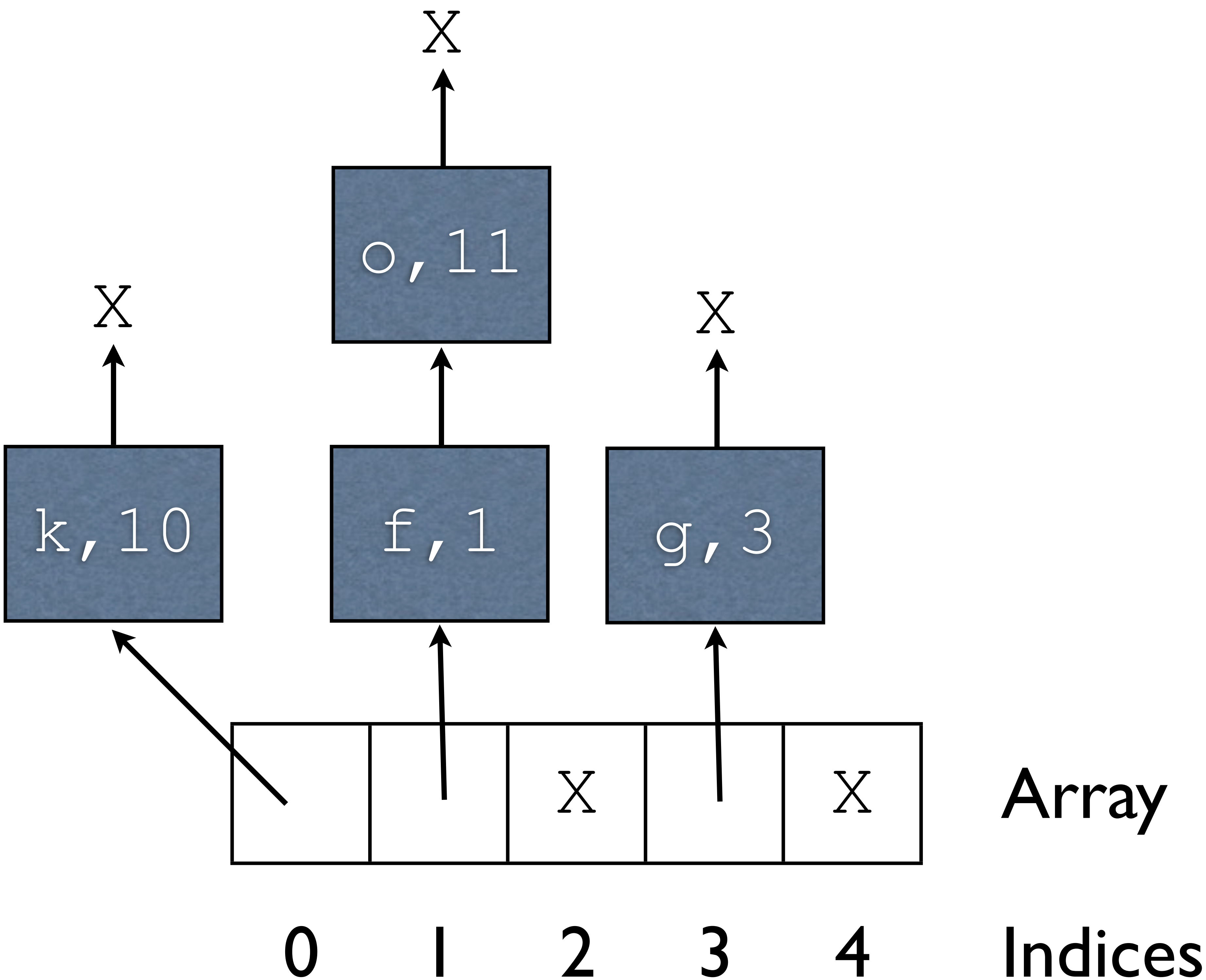


Example

`lookup(11)`



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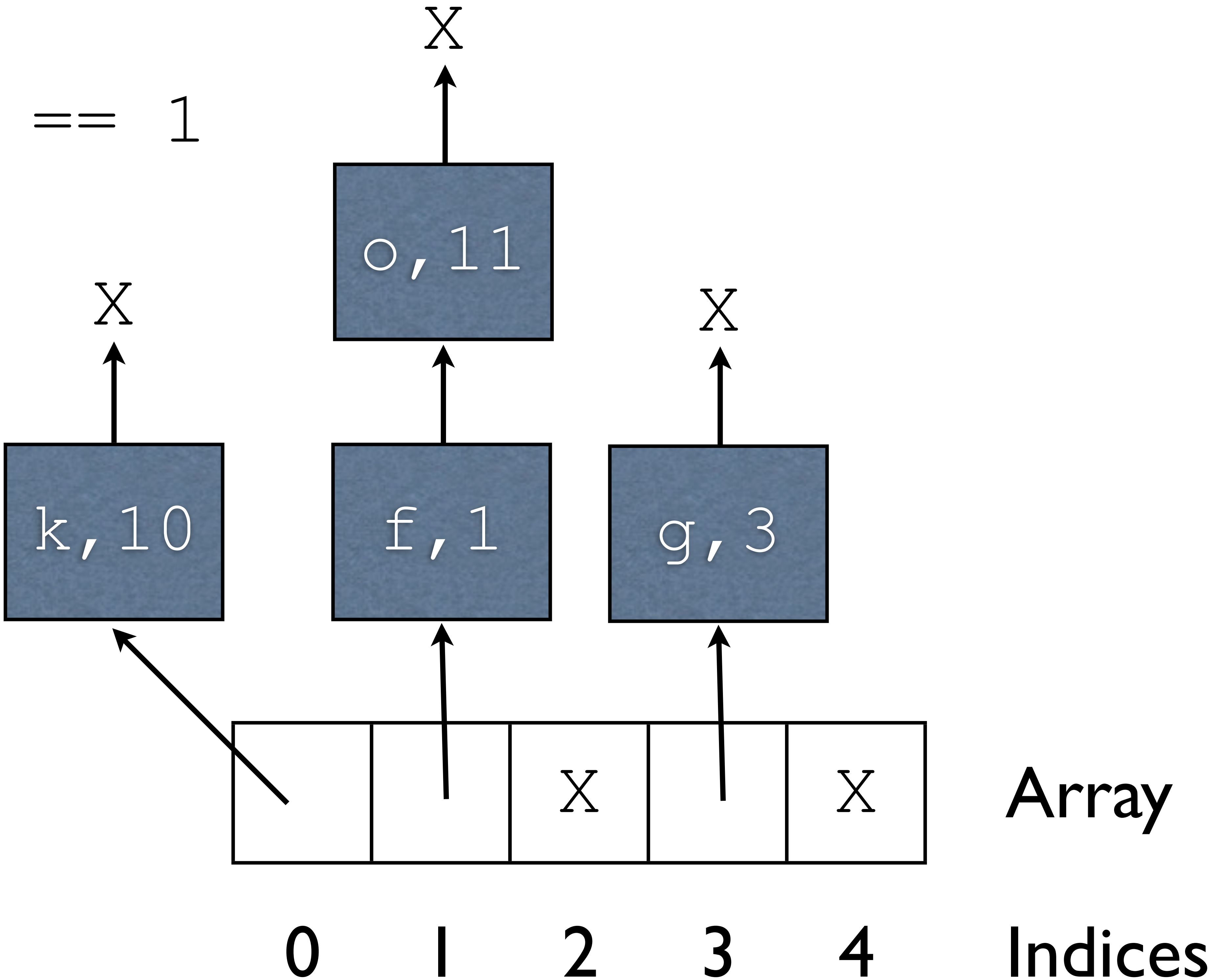

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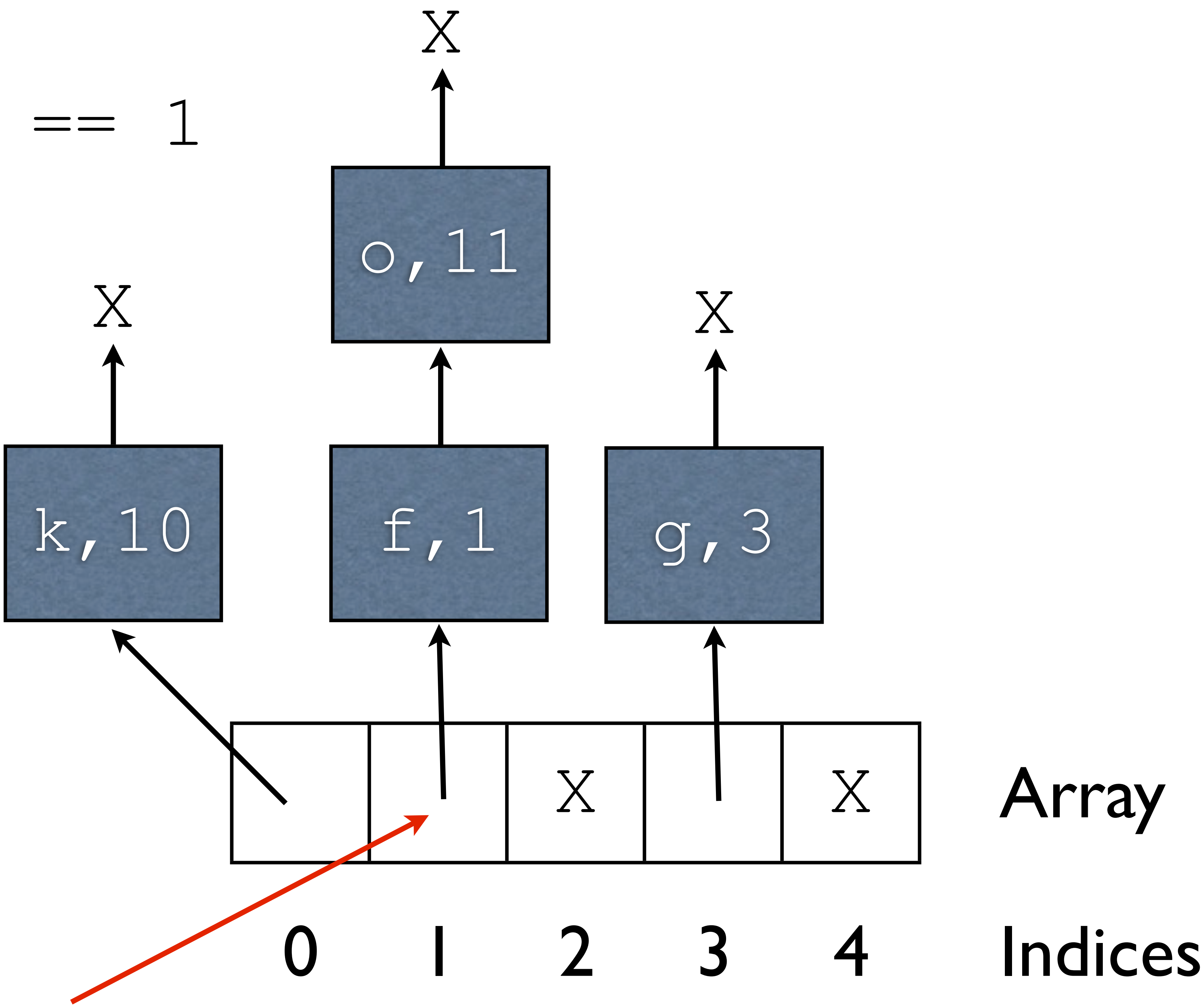

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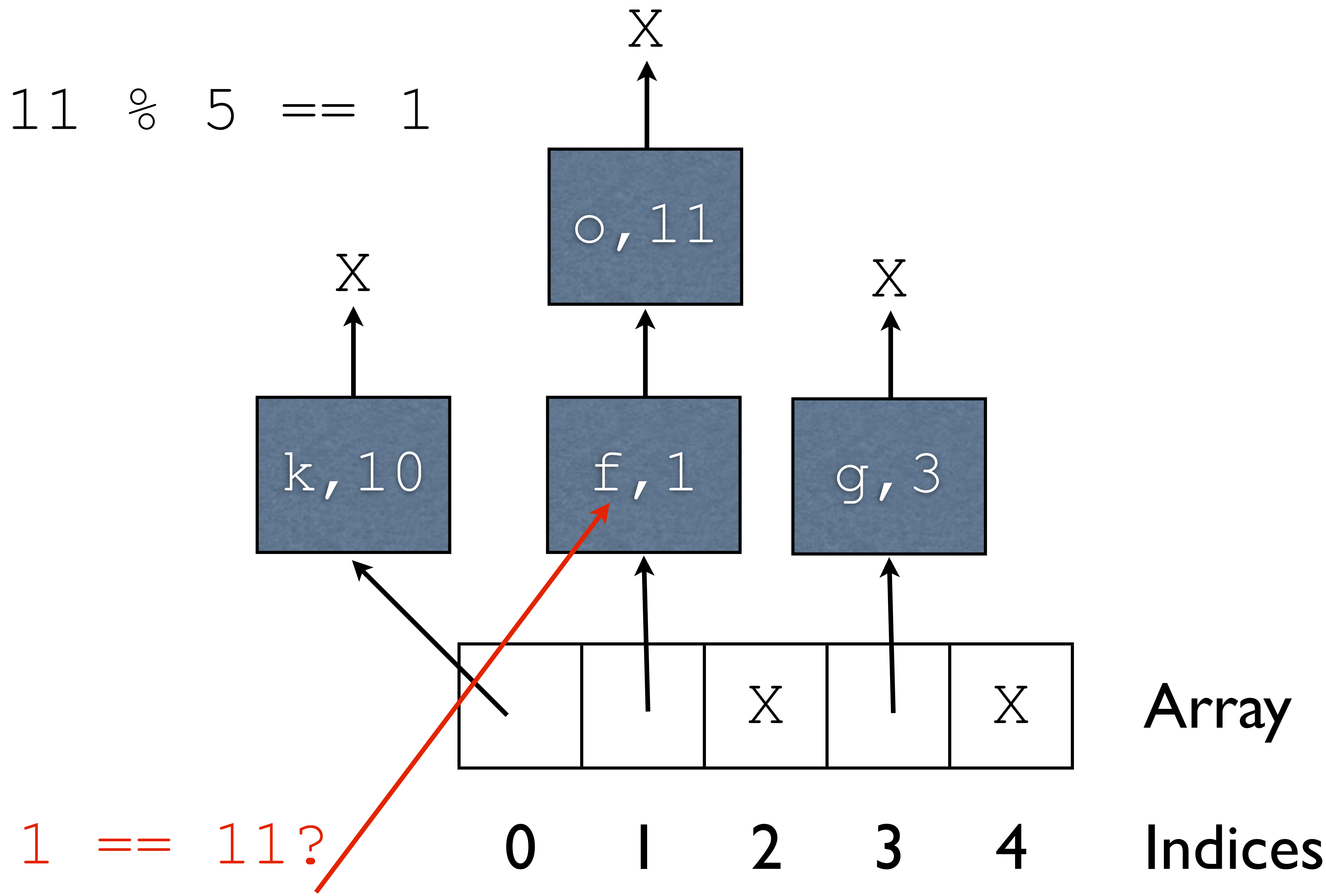

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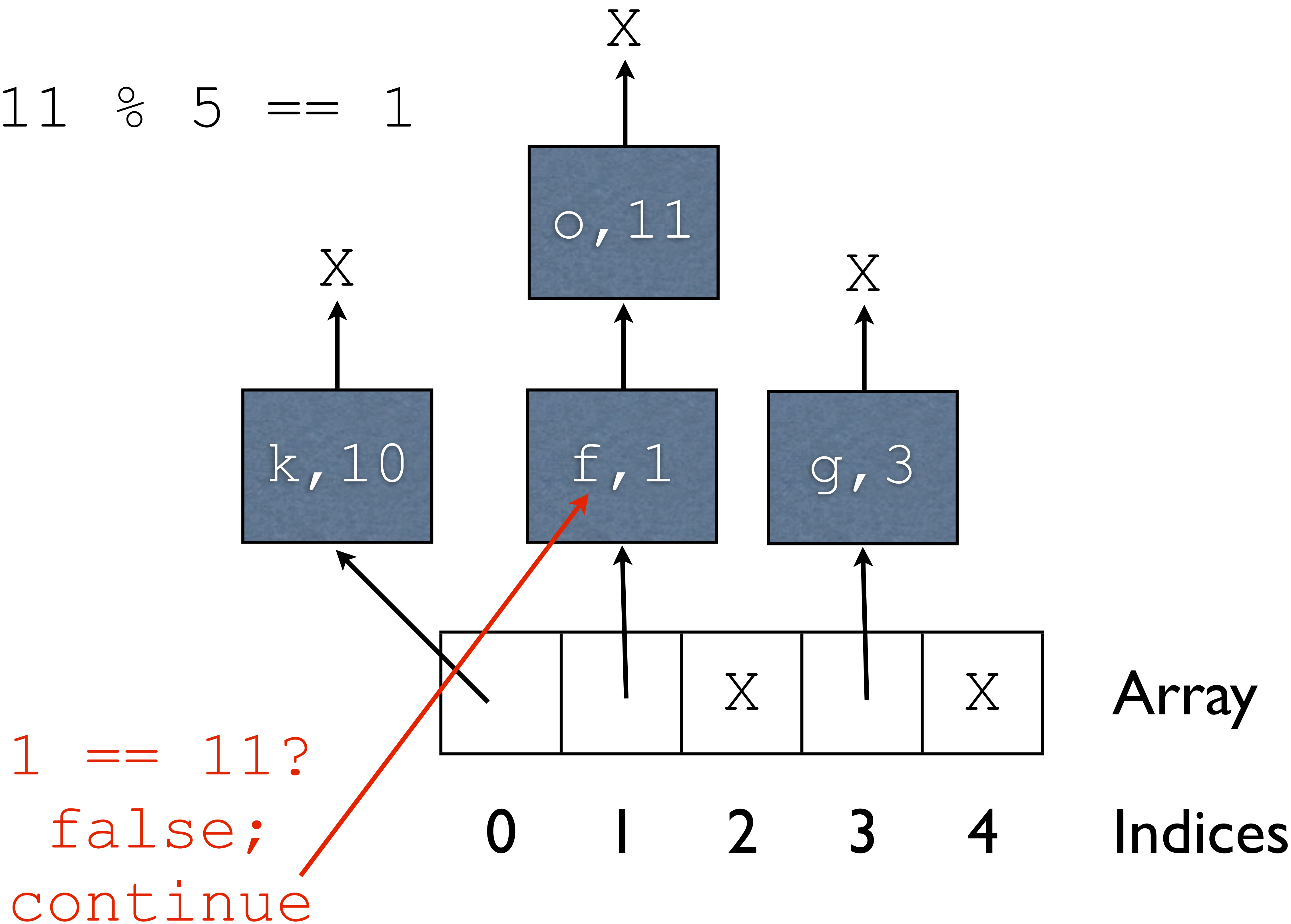

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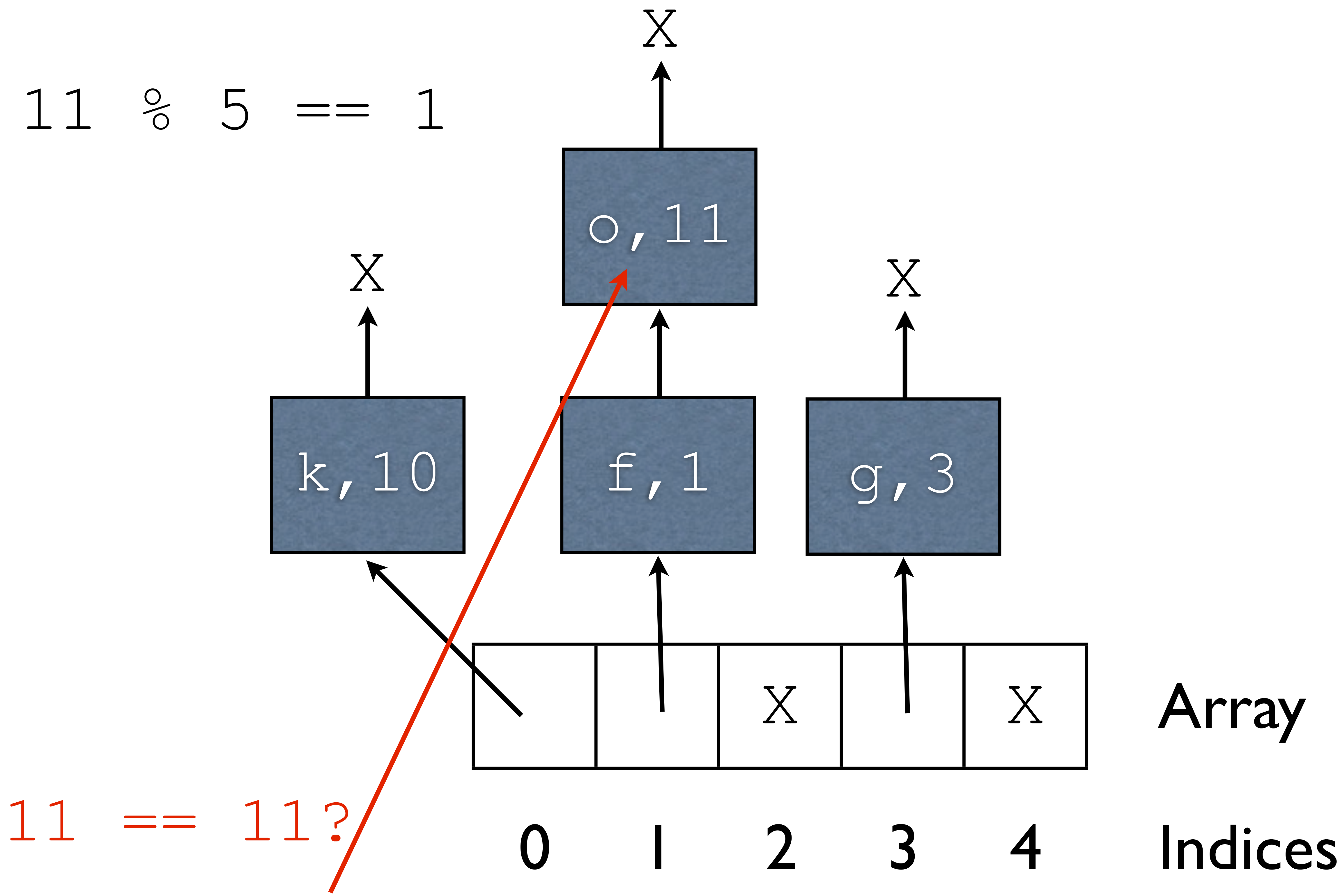

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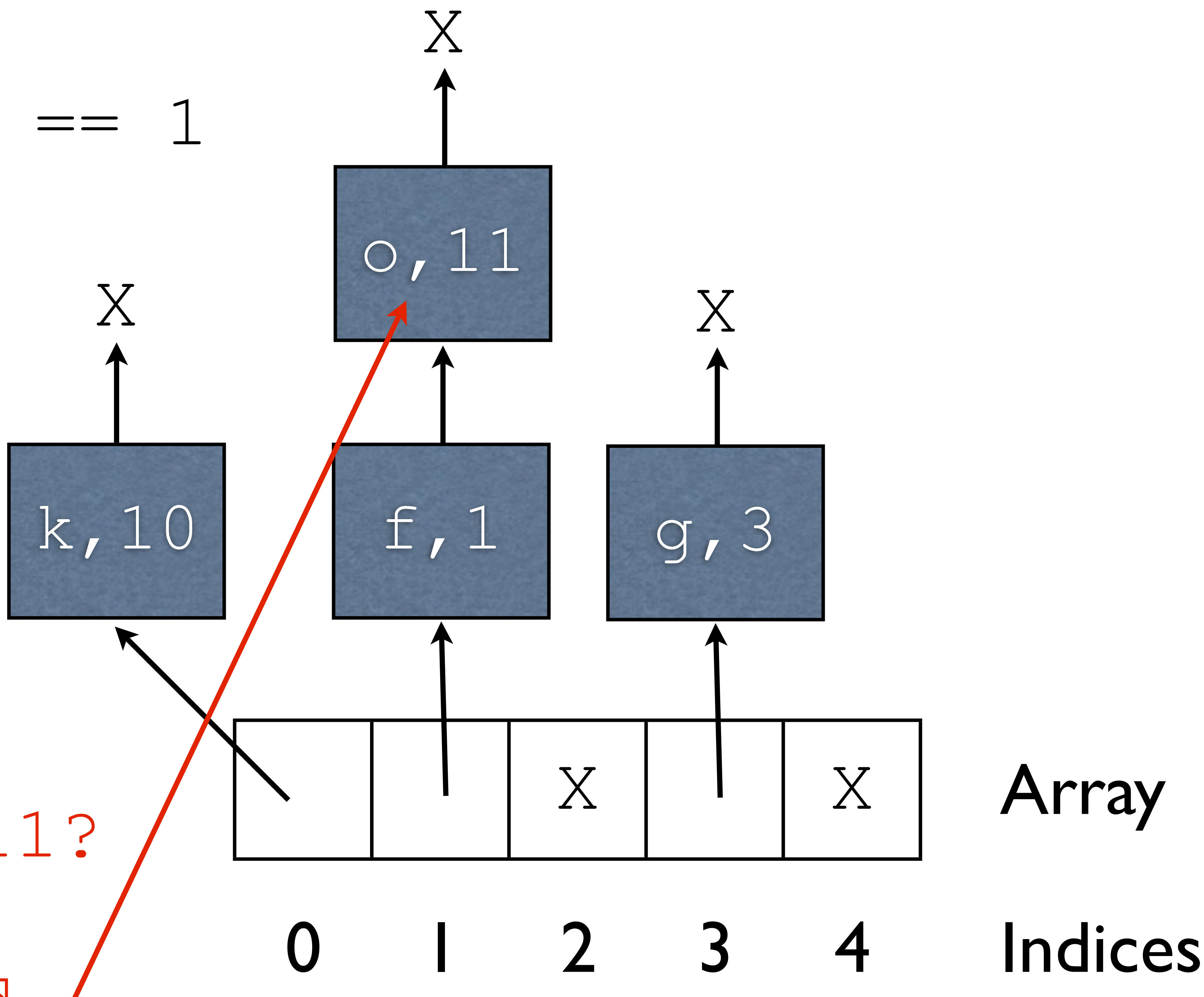
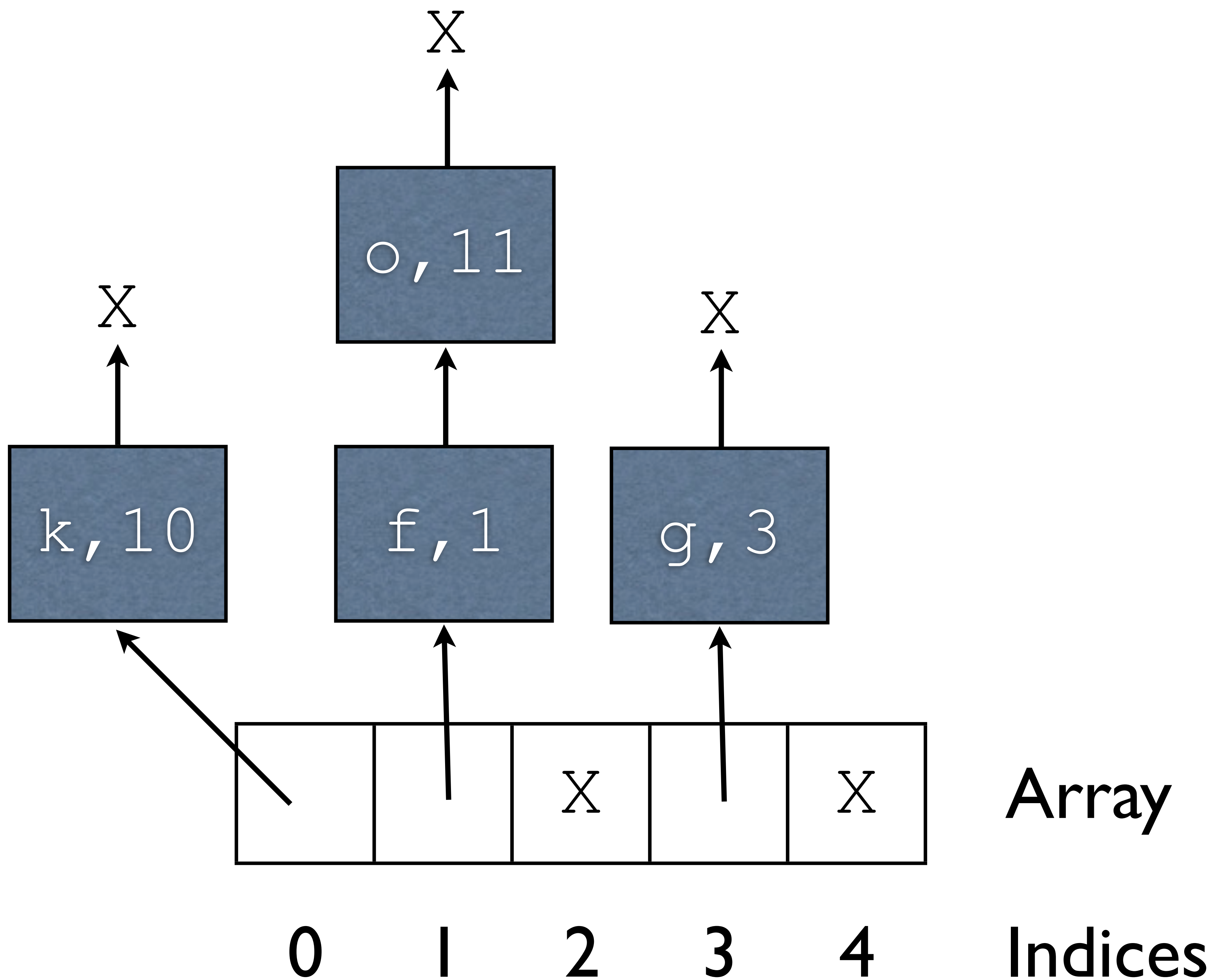


`11 == 11?`
`true;`
`found`

Example

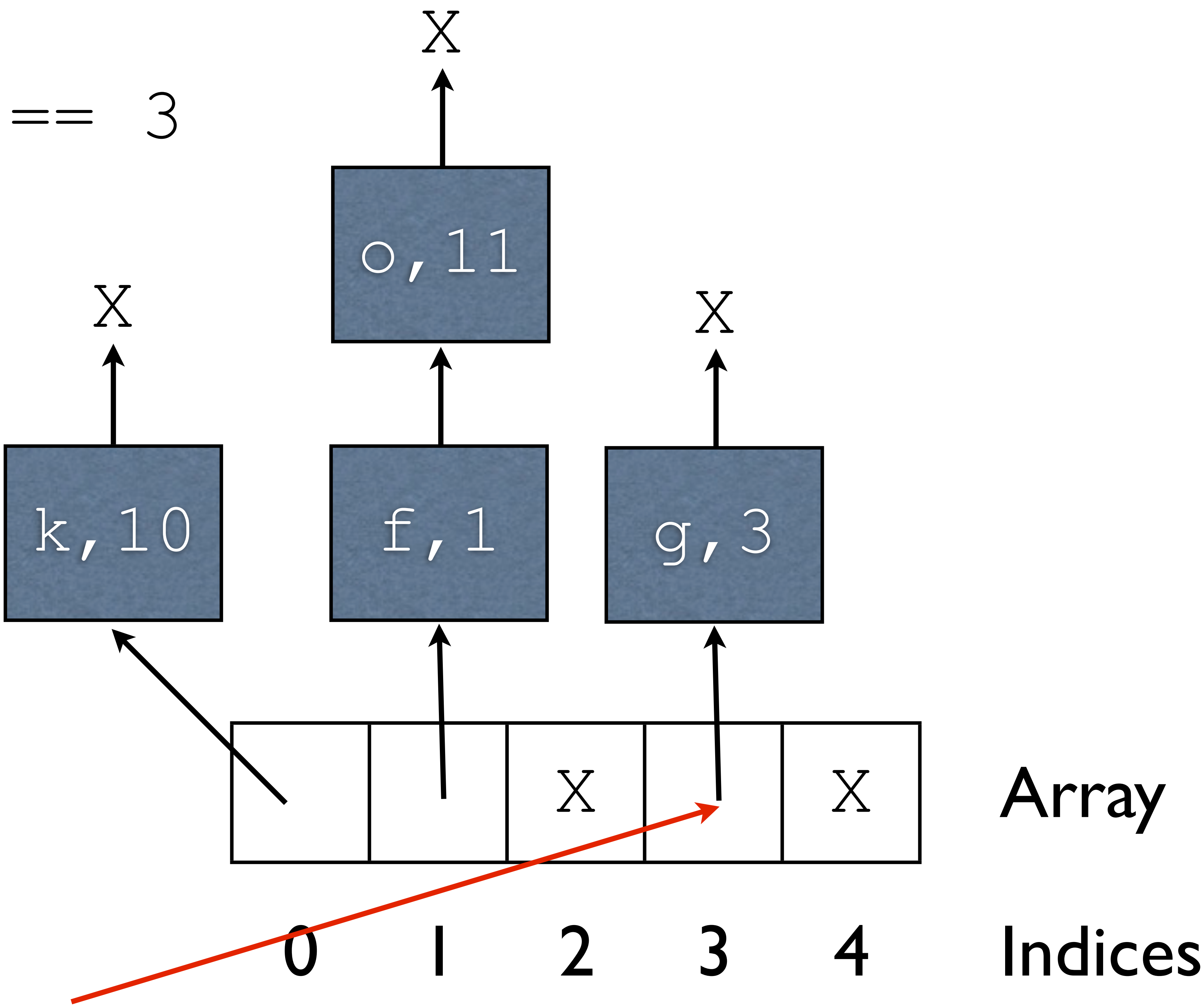
`lookup(8)`



Example

`lookup(8)`

`8 % 5 == 3`



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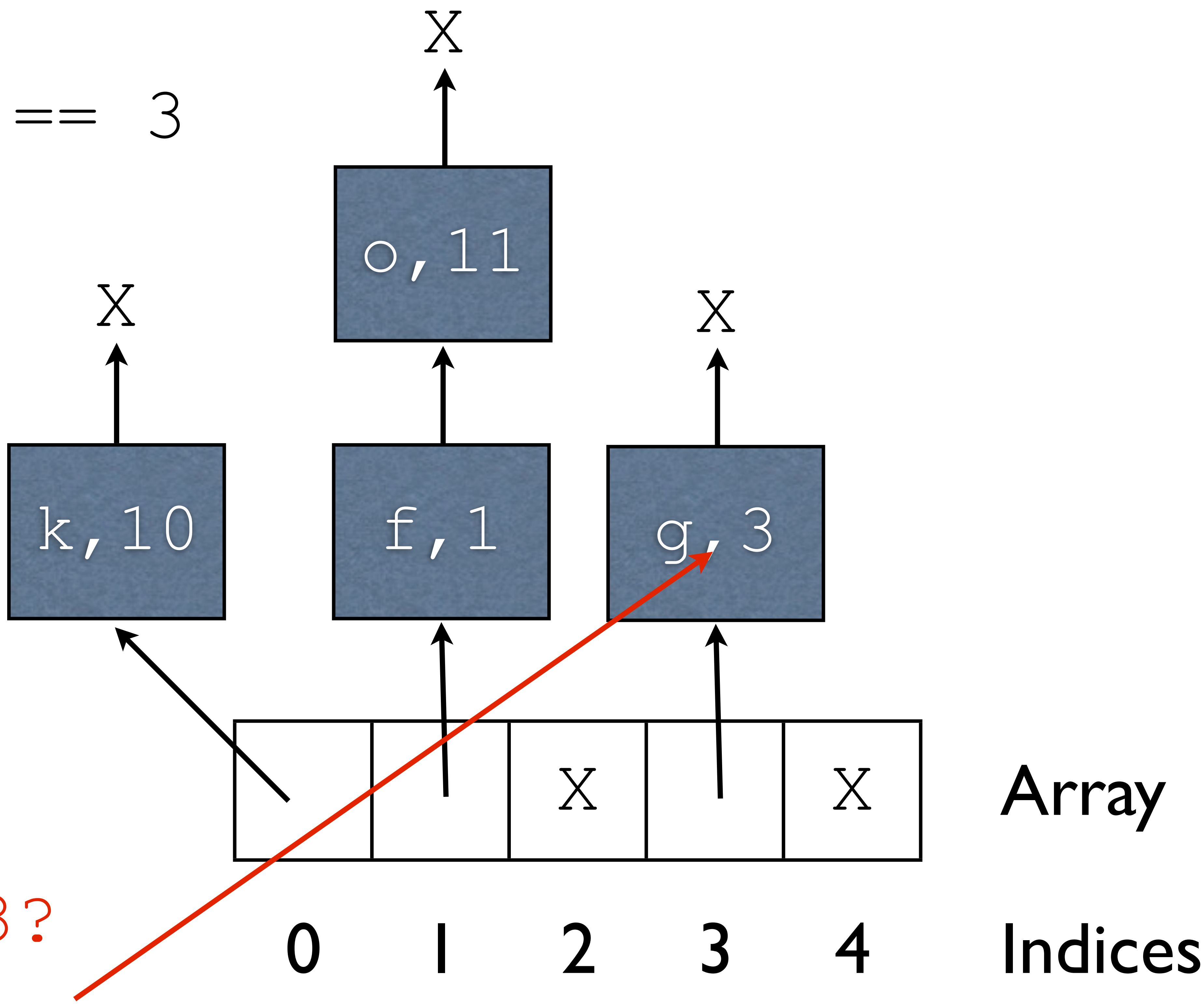

`8 == 3?`

Example

lookup(8)

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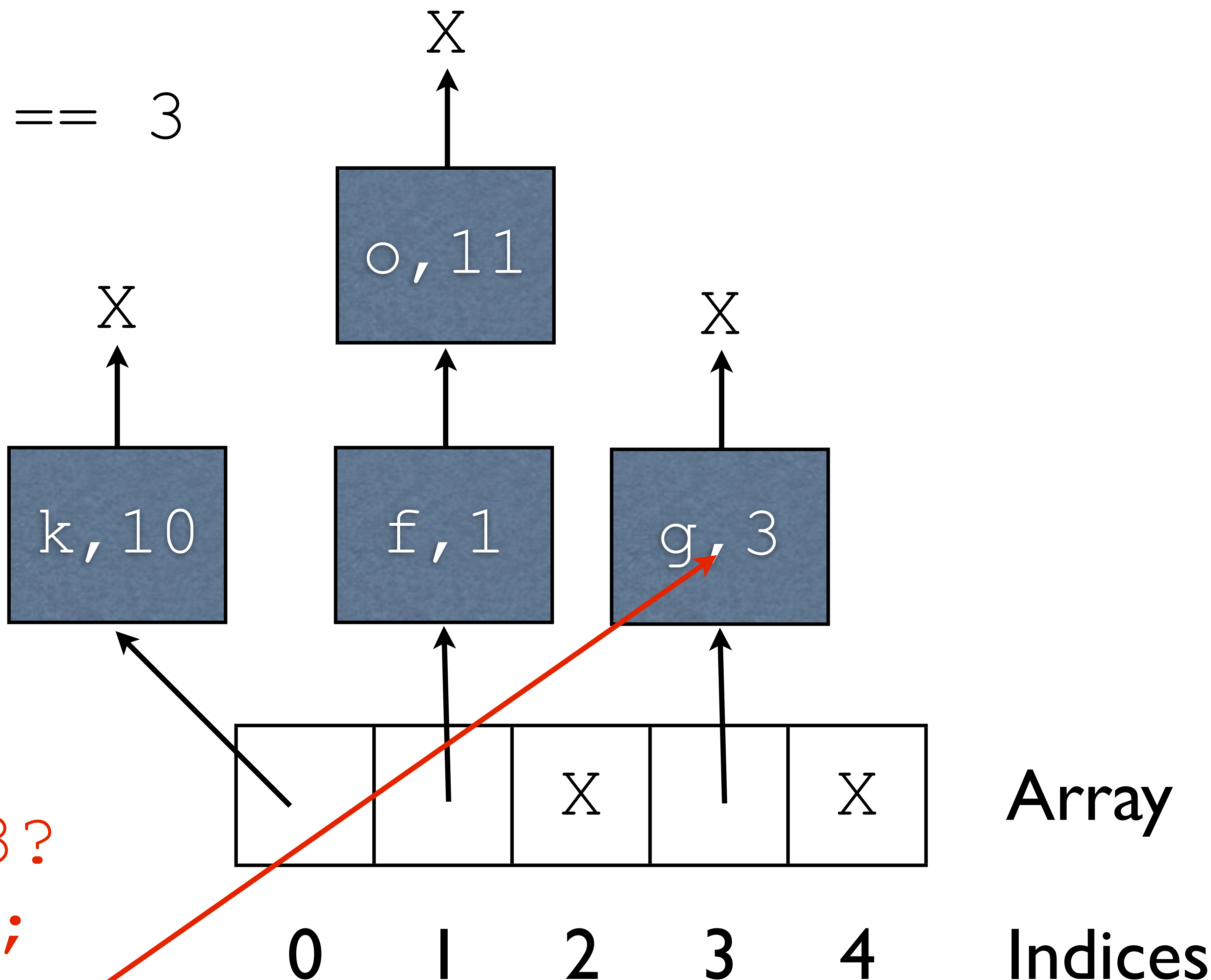
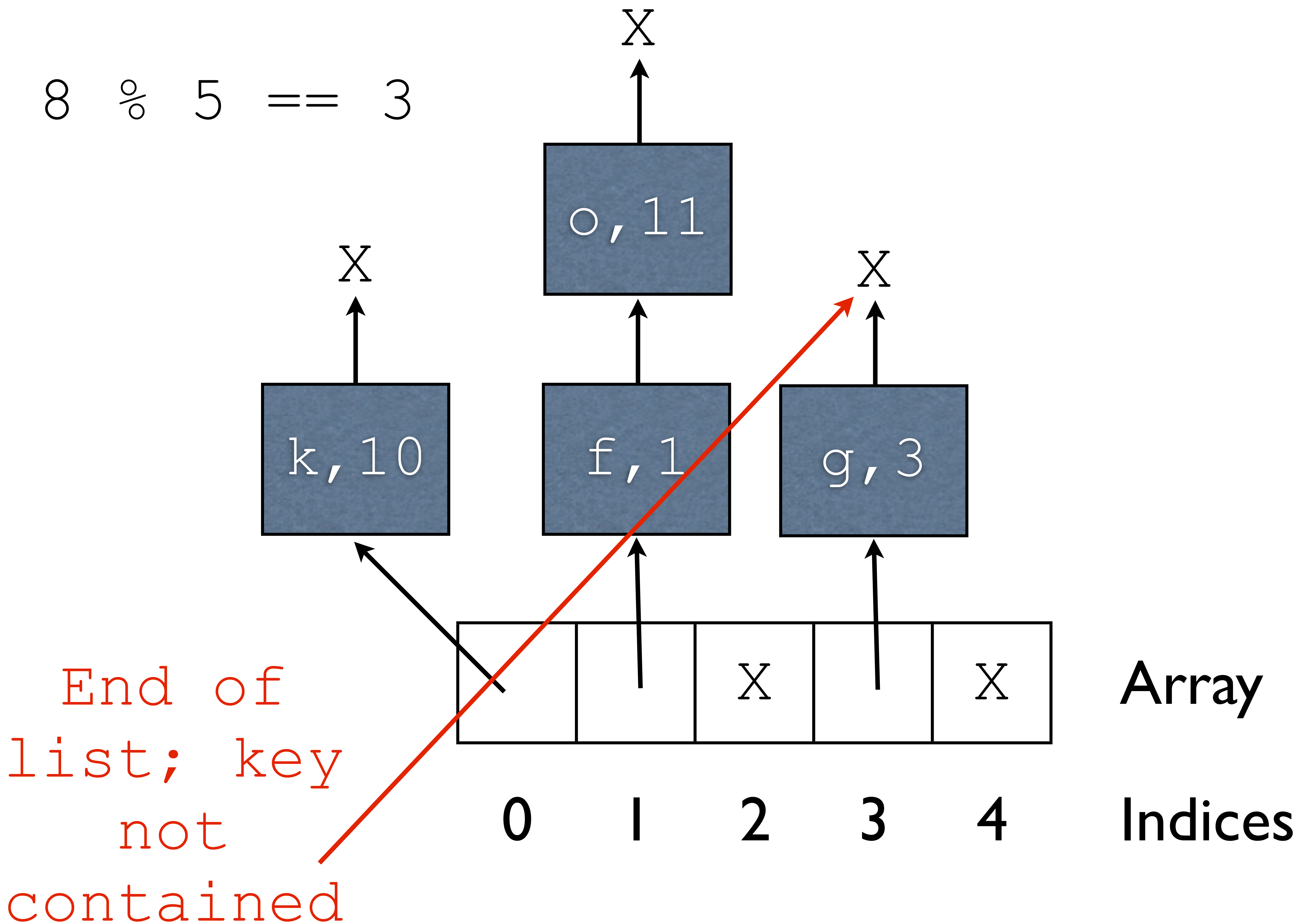


8 == 3?
false;
continue

Example

`lookup(8)`

`8 % 5 == 3`



Lifting Restriction

- To make progress, we had assumed that keys were positive integers
- How might we extend this to arbitrary keys?

Lifting Restriction

- To make progress, we had assumed that keys were positive integers
- How might we extend this to arbitrary keys?
  - Idea: an alternative numeric representation for everything which behaves as a key

Hash Codes

- A way of getting a numeric representation for some non-numeric data
- We can determine which slot a key goes into based on its hash code

```
int stringHashCode(char* str) {
  int retval = 0;
  for(int x = 0;
      x < strlen(str);
      x++) retval += str[x];
  return retval;
}
```

On Performance

- What time complexity do lookups and additions have?

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  - `O(N)`! Worse than the `O(log(N))` we were trying to beat!
- Why is this happening?

On Performance

- What time complexity do lookups and additions have?
  - `O(N)`! Worse than the `O(log(N))` we were trying to beat!
- Why is this happening?
  - Worst case, all keys end up in the same slot (bucket), and this degrades into a linked list

Degradation

- What circumstances make it more likely that a hash table turns into a linked list?

Degradation

- What circumstances make it more likely that a hash table turns into a linked list?
  - Small array - more keys compete for fewer slots (buckets)
  - Hash function claims the majority of the keys are in the same bucket, e.g. `return 0;`

Small Array

- How can we address the issue with the array being small?

Small Array

- How can we address the issue with the array being small?
  - Initial huge allocation: wastes space
  - Dynamically reallocate and redistribute when we get too large: complex and resizing is expensive (common in practice)

Hash Function

- How can we address the issue with the hash function putting everything into the same bucket?

Hash Function

- How can we address the issue with the hash function putting everything into the same bucket?
  - Build a better hash function

```
str[0]*31^(len-1) + str[1]*31^(len-2)
        + ... + str[len-1]
```

Time Complexity

- What are the time complexities after adjusting for the small array issue and improving the hash function?

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  - Still `O(N)`! We didn't change anything in the worst case!

Best-Case Time Complexity

- What is a best-case scenario? What sort of time complexity do we have in this best-case scenario?

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- What is a best-case scenario? What sort of time complexity do we have in this best-case scenario?
  - Each bucket contains at most one entry
  - Constant time - `O(1)`

In Practice

- With a relatively good hash function, in practice, hash tables perform in constant time, despite the `O(N)` worst-case complexity
  - Worst-case complexity only gives you part of the picture
- A little experiment with ~300,000 entries showed that most 95% of buckets had between 0-2 entries, and had at most 7